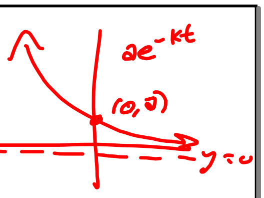
65. Under certain circumstances a rumor spreads according to the equation

$$p(t) = \frac{1}{1 + ae^{-kt}} \xrightarrow{t \to \infty} 1$$



where $p(t)$ is the proportion of the population that has heard the rumor at time t and a and k are positive constants. [In Section 9.4 we will see that this is a reasonable model for $p(t)$.]

(a) Find $\lim_{t\to\infty} p(t)$. $= 1$

(b) Find the rate of spread of the rumor.

(c) Graph p for the case $a = 10$, $k = 0.5$ with t measured in hours. Use the graph to estimate how long it will take for 80% of the population to hear the rumor.

(b) $\frac{d}{dt}\left[\frac{1}{1+ae^{-kt}}\right] = \frac{0(1+ae^{-kt}) - 1(ae^{-kt}\cdot(-k))}{(1+ae^{-kt})^2}$

$= \frac{+kae^{-kt}}{(1+ae^{-kt})^2}$

(c) $\frac{1}{1+10e^{-.5t}}$ very tough by hand. Recommend technology, but old-school: chain.

$$p''(t) = \frac{(-k^2ae^{-kt})(1+ae^{-kt})^2 - kae^{-kt}(2\cancel{(1+ae^{-kt})})(\boxed{-k}ae^{-kt})}{(1+ae^{-kt})^{4 \to 3}}$$

$$= \frac{-k^2ae^{-kt}[(1+ae^{-kt}) - 2ae^{-kt}]}{(\quad)^3}$$

$$= \frac{-k^2ae^{-kt}[1+ae^{-kt} - 2ae^{-kt}]}{(\quad)^3}$$

$$= \frac{-k^2ae^{-kt}[1-ae^{-kt}]}{(1+ae^{-kt})^3}$$

SET 0 →

$p''(0) = -k^2a(1-a)$ / stuff

$= \frac{-.25(10)[1-10]}{\text{stuff}}$

$= \frac{+}{+} = +$

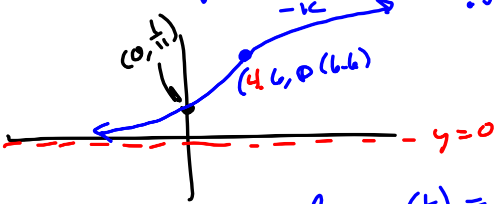
$ae^{-kt} = 1$

$e^{-kt} = \frac{1}{a}$

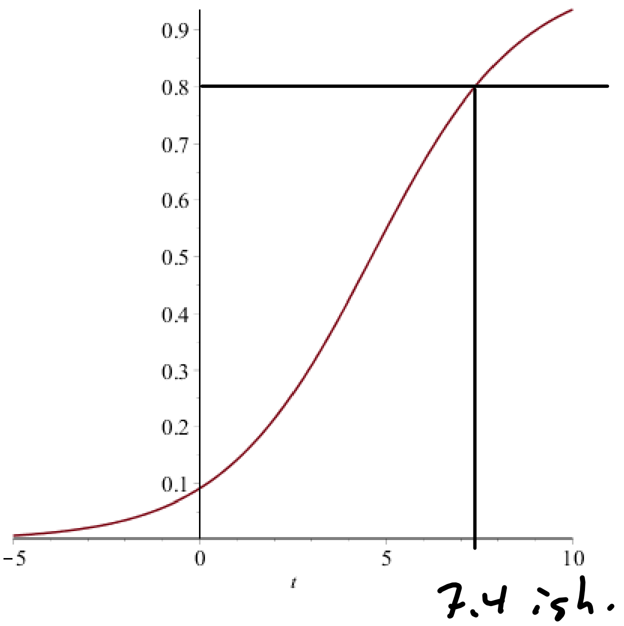
$-kt = \ln\left(\frac{1}{a}\right)$

$t = \frac{\ln\left(\frac{1}{a}\right)}{-k} = \frac{\ln\left(\frac{1}{10}\right)}{-.5} \approx \boxed{4.6057}$

$p(t) = \frac{1}{1+10e^{-.5t}}$



$\lim_{t\to-\infty} p(t) = \lim_{t\to-\infty} \frac{1}{1+ae^{-kt}}$ "=" "$\frac{1}{\infty}$" = "0"



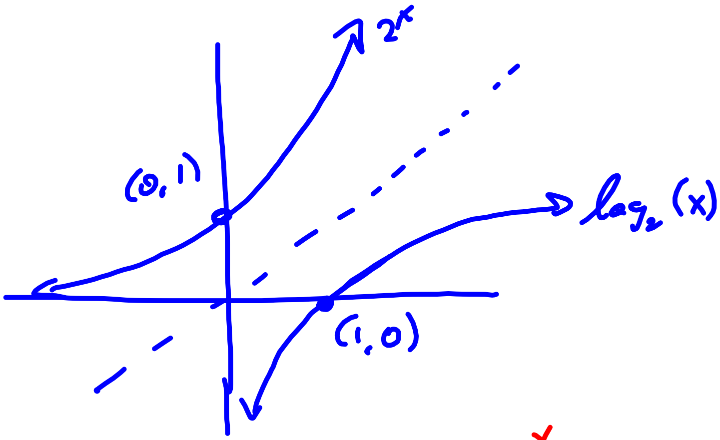
Part c:

$t \approx 7.378$
when $p(t) \approx 80\%$

Differentiate

$$f(x) = \left(2 + \sin(3x^2 + 2x) - 7x\right)^5$$

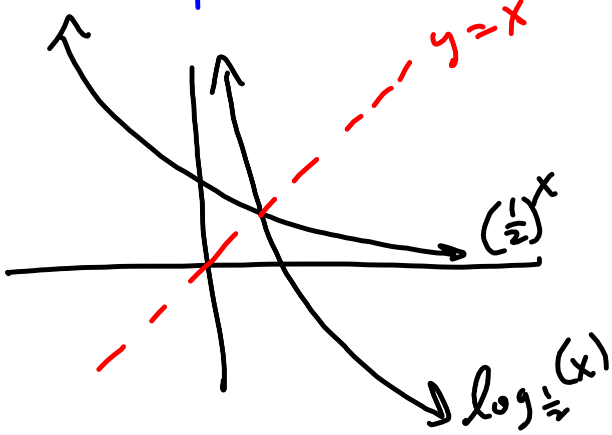
$$\Rightarrow f'(x) = 5\left(2 + \sin(3x^2+2x) - 7x\right)^4 \cdot$$
$$\cdot \left(\cos(3x^2+2x)(6x+2) - 7\right)$$



FACT:

If f is increasing, so is f^{-1}

If f is decreasing so is f^{-1}



$$\frac{d}{dx}\left[e^x\right] = e^x \qquad \frac{d}{dx}\left[e^{g(x)}\right] = g'(x)\,e^{g(x)}$$

$$\frac{d}{dx}\left[\ln(x)\right] = \frac{1}{f'(f^{-1}(x))} = \frac{1}{e^{\ln(x)}} = \frac{1}{x}$$

$$f(x) = e^x$$
$$f^{-1}(x) = \ln(x)$$

$$\frac{d}{dx}\left[\ln(g(x))\right] = \frac{g'(x)}{g(x)}$$

will derive at more length if you wish.

$$\int \frac{1}{x}\,dx = \ln(x) + c \qquad (x > 0)$$

$$\int \frac{g'(x)}{g(x)}\,dx = \int \frac{du}{u} = \ln(u) + C$$