§6.1 #28 Find the inverse.

$y = 2x^2 - 8x = 2x(x-4)$

$2y^2 - 8y = x, \quad x \geq 2$

$y^2 - 4y + 2^2 = \frac{1}{2}x + 4$

$(y-2)^2 = \frac{1}{2}x + 4$

$\sqrt{(y-2)^2} = \sqrt{\frac{1}{2}x+4}$

$|y-2| = \sqrt{\frac{1}{2}x+4}$

$y - 2 = \pm\sqrt{\frac{1}{2}x+4}$

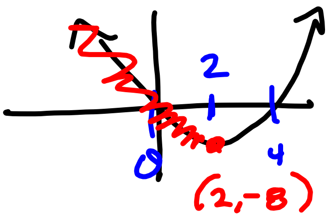
$y = \pm\sqrt{\frac{1}{2}x+4} + 2$

$= \pm\sqrt{\frac{1}{2}(x+8)} + 2$

$\sqrt{3^2} = 3$

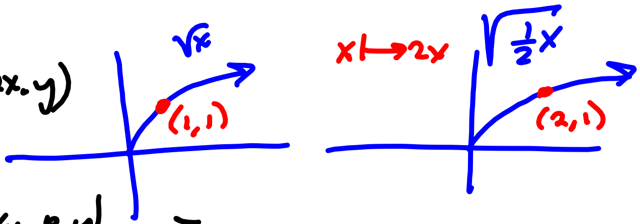
$\sqrt{(-3)^2} = 3$

$\sqrt{x^2} = \begin{cases} x \\ -x \end{cases} = |x|$

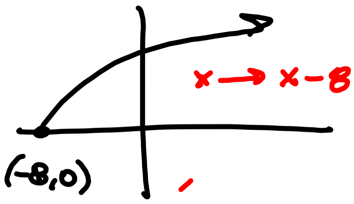


$f(\frac{1}{2}x)$

$(x,y) \mapsto (2x, y)$

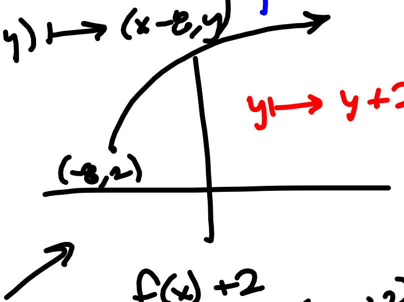


$\sqrt{\frac{1}{2}(x+8)}$



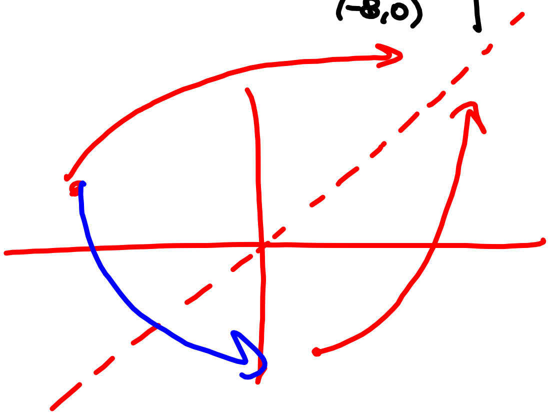
$f(x+8)$

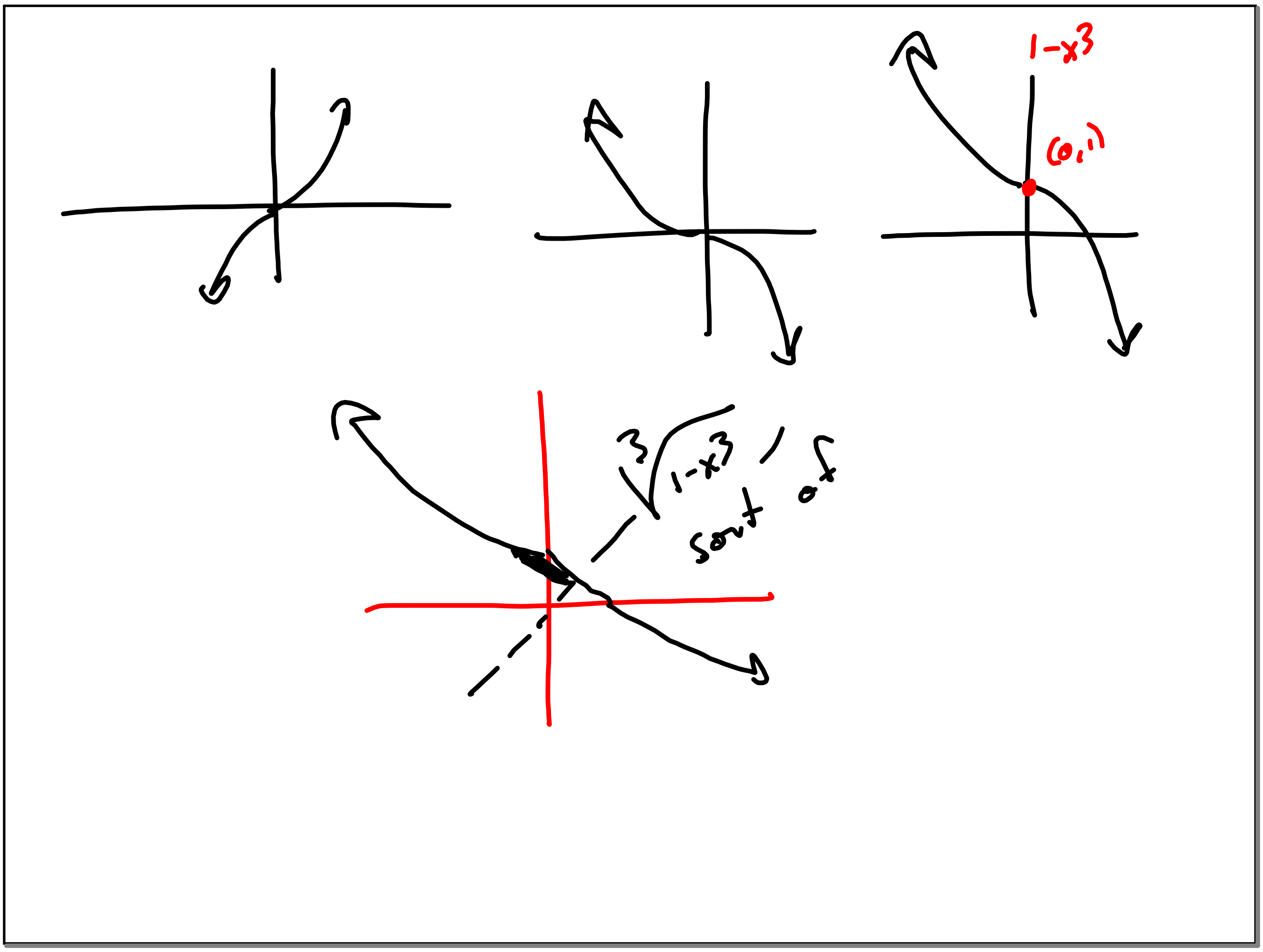
$(x,y) \mapsto (x-8, y)$



$f(x)+2$

$(x,y) \mapsto (x, y+2)$





$1-x^3$
(0,1)
$\sqrt[3]{1-x^3}$
sort of

$f(x) = x^2 - 2x$ is not 1-to-1.

Scratch $f(x_1) = f(x_2) \Rightarrow$

$$x_1^2 - 2x_1 = x_2^2 - 2x_2$$

$$x_1^2 - 2x_1 - x_2^2 + 2x_2 = 0$$

$$x_1^2 - x_2^2 - 2x_1 + 2x_2 = 0$$

$$(x_1 - x_2)(x_1 + x_2) - 2(x_1 - x_2) = 0$$

$$(x_1 - x_2)[x_1 + x_2 - 2] = 0 \Rightarrow$$

$x_1 - x_2 = 0$ OR $x_1 + x_2 - 2 = 0$

Is the key to proof by counterexample.

$f(x) = x^2 - 2x$

$f(5) = 25 - 10 = 15$

$f(-3) = 9 + 6 = 15$

$x_1 + x_2 - 2 = 0$

$x_2 = -x_1 + 2$

Let $x_1 = 5. \Rightarrow$

$x_2 = -5 + 2 = -3$

Proof $f(5) = f(-3) = 15$ ■

$x_2 = x_1$

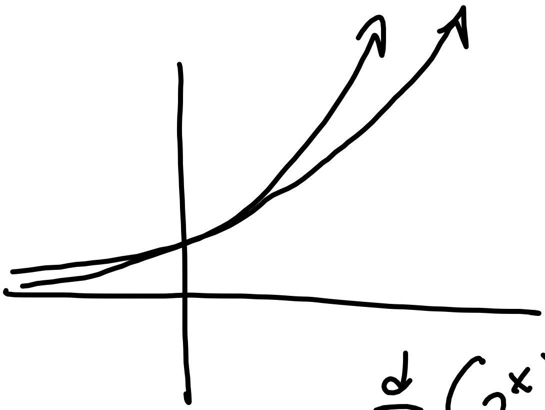
$x_2 = -x_1 + 2$ } 2 ways.

⋮

$x_1 = x_2$

$f(x) = e^x$ $\qquad e \approx 2.7128 \ldots$

$f'(x) = e^x$ $\qquad 2 < e < 3$



2^x is less steep than it is tall $\quad \ln(2)\cdot 2^x < 2^x$

3^x is steeper than it is tall $\quad \frac{d}{dx}[3^x] = \ln(3)\cdot 3^x > 3^x$

$$\frac{d}{dx}(2^x) =$$

$$2^x = e^{\ln(2^x)} = e^{x\cdot\ln(2)} = e^{(\ln(2))x} \Rightarrow \frac{d}{dx}\left[e^{\ln(2)\cdot x}\right] = \left(e^{\ln(2)\cdot x}\right)\ln(2)$$

$$= e^{\ln(2^x)}\cdot\ln(2) = 2^x\cdot\ln(2) = \ln(2)\cdot 2^x$$

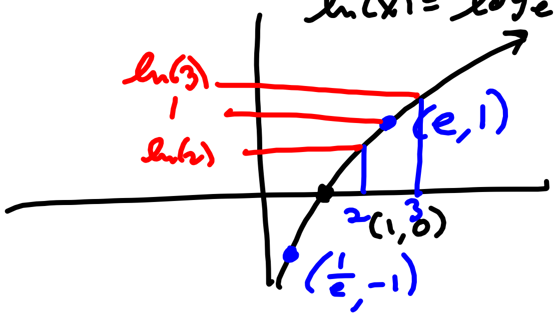
Chain Rule

$$\frac{d}{dx}[f(g(x))] = f'(g)\cdot g'(x)$$

$$u = f(g(x)) \Rightarrow \frac{du}{dx} = \frac{du}{dg}\cdot\frac{dg}{dx}$$

$$\frac{d}{dx}\left[\sin(x^2+2x)\right] = \cos(x^2+2x)\cdot(2x+2)$$

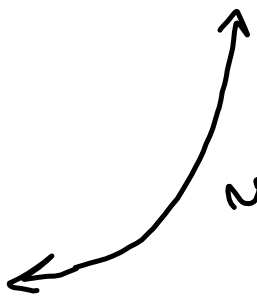
$\ln(x) = \log_e(x)$



$$\frac{1}{e} = e^{-1}$$

Early part of 6.2 video is mainly trying to convince you that

e^x is a continuous function.



No gaps.

Jan 17-12:03 PM