Steve Mills, MAT 202

Calculus II

CALC II : $\boxed{\text{Advanced Integration Techniques}}$

Series → Your calculator uses series to calculate sine, cosine, ..

Polar Coords, funcs, graphs. (Taylor Polynomial)

Start the course : Exponentials & Logarithmics.

$3^x = 7$ $\quad f(x) = 3^x$

$f^{-1}(x) = \log_3(x)$

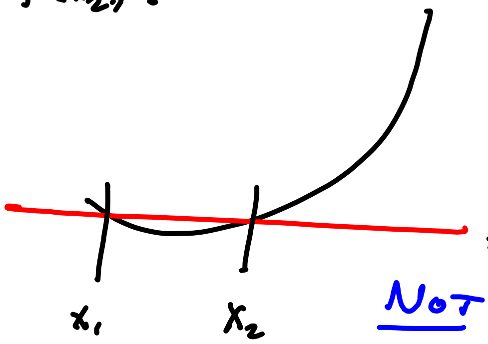
$\log_3(3^x) = \log_3(7)$

$x = \log_3(7) = \dfrac{\ln(7)}{\ln(3)}$

$f^{-1}(f(x)) = x$

§6.1 Inverse Functions.

f is 1-to-1 means if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$.



$f(x_1) = f(x_2)$

NOT 1-to-1.

$x_1 \neq x_2$, but $f(x_1) = f(x_2)$

$f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$, if f is 1-to-1.

Prove $f(x) = \dfrac{x-1}{x+3}$ is 1-to-1.

Suppose $f(x_1) = f(x_2)$

$$\frac{x_1 - 1}{x_1 + 3} = \frac{x_2 - 1}{x_2 + 3} \qquad \text{(Isolate the "}x_1\text{")}$$

$$(x_1 - 1)(x_2 + 3) = (x_2 - 1)(x_1 + 3)$$

$$\cancel{x_2 x_1} + 3x_1 - x_2 \cancel{-3} = \cancel{x_2 x_1} + 3x_2 - x_1 \cancel{-3}$$

$$3x_1 - x_2 = 3x_2 - x_1$$

$$4x_1 = 4x_2$$

$$x_1 = x_2 \quad \blacksquare$$

Derivative of the inverse function

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

Handy for messy f^{-1}'s.

Technique

$(f^{-1})'(a)$:

Find b, such that

$f(b) = a$

Then $(f^{-1})'(a) = \frac{1}{f'(b)}$

Example Find $(f^{-1})'(3)$

for $f(x) = x^5 + x^3 + x$

Solve $f(x) = 3 \Rightarrow x = 1 = f^{-1}(3)$

FIND the b

$f'(x) = 5x^4 + 3x^2 + 1$

$f'(1) = 5 + 3 + 1 = 9$

$$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(1)} = \frac{1}{9}$$

Proof of the result.

$$(f^{-1})'(x) = \lim_{h \to 0} \frac{(f^{-1})(x+h) - (f^{-1})(x)}{h}$$

Now, $(f^{-1})'(a) = \lim_{h \to 0} \frac{(f^{-1})(a+h) - (f^{-1})(a)}{h}$

$$= \lim_{x \to a} \frac{(f^{-1})(x) - (f^{-1})(a)}{x-a}$$ Same Deal.

$x = f(y)$ $\quad$ $f(b) = a$

Let $y = f^{-1}(x)$ and $b = f^{-1}(a)$.

Then (Drop the limit.):

$\frac{2}{3} = \frac{1}{\left(\frac{3}{2}\right)}$

$$\frac{y-b}{f(y)-f(b)} = \frac{1}{\frac{f(y)-f(b)}{y-b}} \xrightarrow[y \to b]{x \to a}$$

since f is cnt^s.

$$\xrightarrow{y \to b} \frac{1}{f'(b)}$$