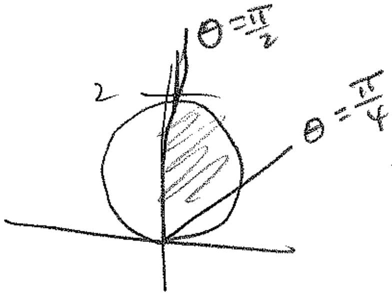
202 § 11.5 #s 2, 5, 8, 10 - 12, 16, 29, 23, 30

(2) Find area of region bdd by $r = 2\sin(\theta)$ $\theta \in [\frac{\pi}{4}, \frac{\pi}{2}]$



$$\text{Area} = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 4\sin^2\theta \, d\theta$$

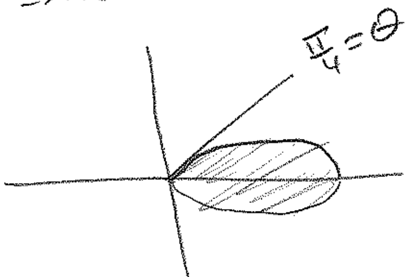
$$= \frac{1}{2}\cdot 4 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1-\cos(2\theta)}{2} \, d\theta$$

$$= 2\cdot\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1-\cos(2\theta))\,d\theta = \left[\theta + \frac{1}{2}\sin(2\theta)\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left(\frac{\pi}{2} - \frac{1}{2}\sin\pi\right) - \left(\frac{\pi}{4} - \frac{1}{2}\sin\frac{\pi}{2}\right) = \frac{\pi}{2} - 0 - \frac{\pi}{4} + \frac{1}{2}$$

$$= \boxed{\frac{\pi}{4} + \frac{1}{2}}$$

(5) Inside one leaf of 4-leaved rose $r = \cos(2\theta)$



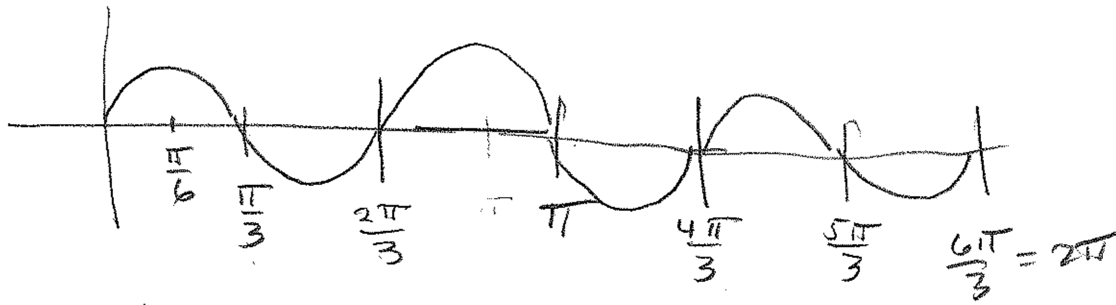
$$2\int_0^{\frac{\pi}{4}} \frac{1}{2}(\cos^2(2\theta))\,d\theta$$

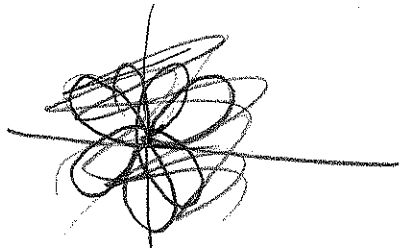
$$= \int_0^{\frac{\pi}{4}} \frac{\cos 4\theta + 1}{2}\,d\theta = \frac{1}{2}\left[\frac{\sin(4\theta)}{4} + \theta\right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}\left[0 + \frac{\pi}{4} - (0+0)\right] = \boxed{\frac{\pi}{4}}$$

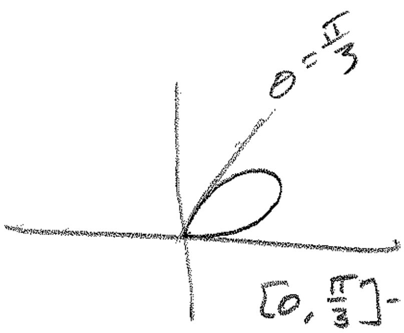
202 §11.5 #s 8, 10–12, 16, 23, 29, 30

⑧ Inside the 6-leaved rose $r^2 = 2\sin(3\theta)$





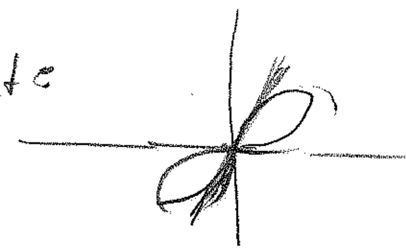
Need to keep
$\sin 3\theta \geq 0$ for r^2 to work.
Then have $\pm$ version, i.e.,
everything gets reflected
thru the origin for the $\pm$ parts.



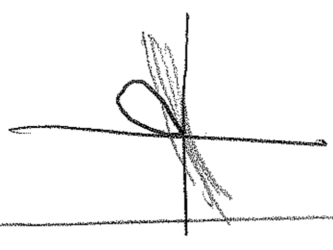
$[0, \frac{\pi}{3}]$-pic.

$r = \sqrt{2\sin 3\theta}$

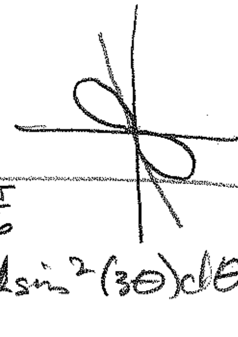
reflect:



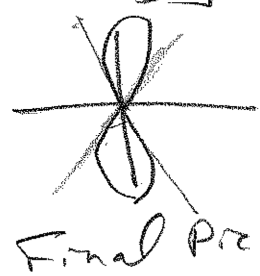
$[\frac{2\pi}{3}, \pi]$-pic



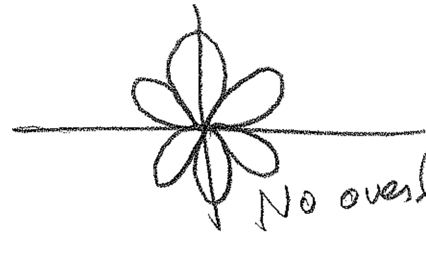
reflect:



$\left[\frac{4\pi}{3}, \frac{5\pi}{3}\right]$-pic:



Final pic:



No overlap.

$$\boxed{\text{Area} = 6 \cdot 2 \cdot \frac{1}{2}\int_0^{\frac{\pi}{6}} \cancel{4}\sin^2(3\theta)\,d\theta}$$

6 leaves

Integral's getting $\frac{1}{2}$-loop

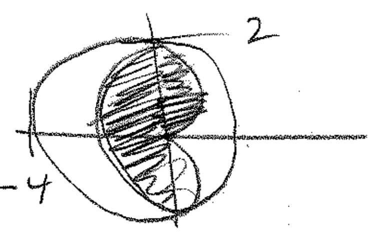
202 §11.5 #s 10–12, 16, 23, 29, 30

#s 9–16 Find the areas

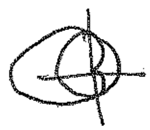
(10) Did it to death in class

(11) $r=2$ & cardioid $r = 2(1-\cos\theta)$

$\frac{1}{2}$ circle $+ 2 \cdot$ 



$$= \frac{\pi (2)^2}{2} + 2\int_0^{\frac{\pi}{2}} \frac{1}{2}\left(4(1-\cos\theta)^2\right) d\theta$$

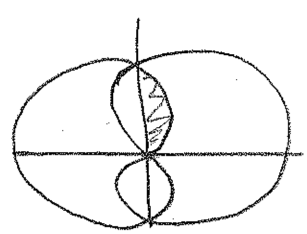


$$= 2\pi + 4\int_0^{\frac{\pi}{2}} (\cos^2\theta - 2\cos\theta + 1)\, d\theta, \text{ etc.}$$

(12) $r = 2(1+\cos\theta)$

$r = 2(1-\cos\theta)$

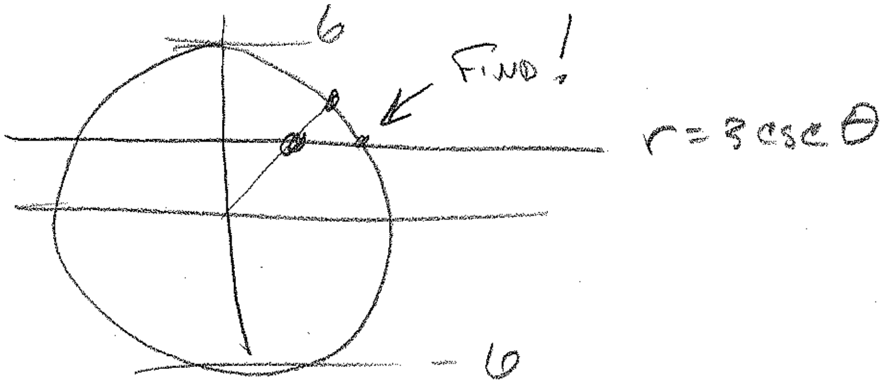
All kinds of symmetry



$$4\int_0^{\frac{\pi}{2}} \frac{(2(1-\cos\theta))^2}{2}\, d\theta$$

202 § 11.5 #5 ~~12~~, 16, 23, 29, 30

~~12~~ 16 Inside $r=6$ & ~~outside r=4~~
above $r=3\csc\theta$



$$3\csc\theta = 6$$
$$\csc\theta = 2$$
$$\sin\theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{6}$$

$$\text{Area} = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(6^2 - (3\csc\theta)^2\right)\, d\theta$$

Guys I need to go home, eat & sleep.
will wipe it out soon