202 §11.4 #s 1, 6, 9, 10, 13, 14, 17, 18, 25, 26

~~#s~~ 1–12 Symmetry, Sketch

1

$r = 1 + \cos\theta$

$r = 1 + \cos(-\theta)$

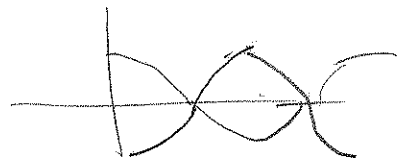
$= 1 + \cos\theta$

$\boxed{x\text{-axis}}$ ✓

$r = 1 + \cos(\pi - \theta)$

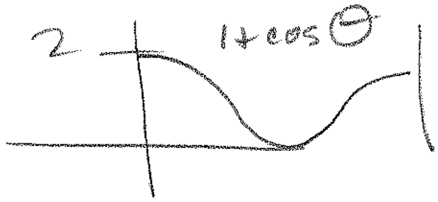
$-r = 1 + \cos(-(\theta - \pi))$

$= 1 + \underbrace{\cos(\theta - \pi)}_{-\cos\theta} =$

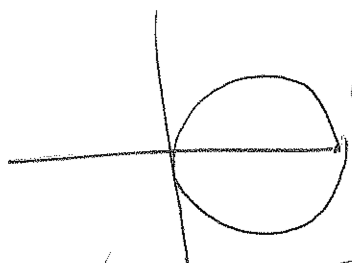


Nope

$-r = 1 + \cos(-\theta)$ $\boxed{\text{No } y\text{-axis}}$ $\boxed{\text{No origin}}$



Not quite



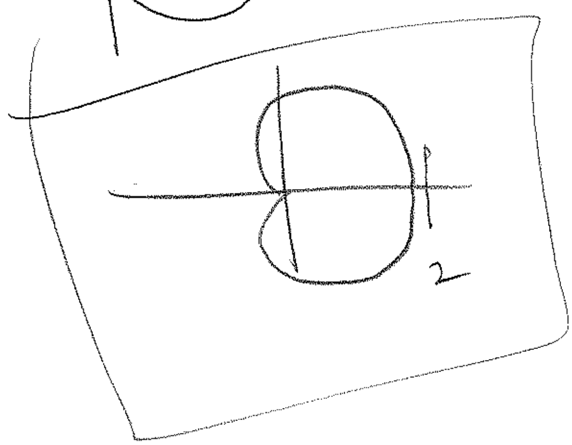
$\sqrt{x^2+y^2} = 1 + \frac{x}{\sqrt{x^2+y^2}}$

$1 = \sqrt{x^2+y^2} + x$

$-x + 1 = \sqrt{x^2+y^2}$

$x^2 - 2x + 1 = x^2 + y^2$

$-2x + 1 = y^2$



202 § 11.4 #s 6, 9, 10, 13, 14, 17, 18, 25, 26

6. $r = 1 + 2\sin\theta$

$r = 1 + 2\sin(-\theta)$? Nope.

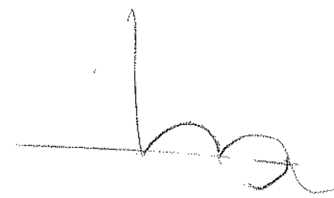
$-r = 1 + 2\sin(\pi - \theta)$?

$-r = 1 - 2\sin(\pi - \theta)$?

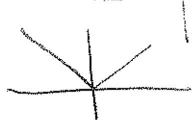
$-r = 1 + 2\sin\theta$

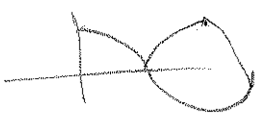
$r = -1 - 2\sin\theta$ NO

x-axis NO



y-axis





~~$r = 1 + 2\cos(\pi - \theta)$?~~

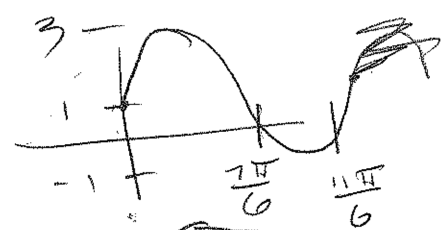
~~$r = 1 + 2\cos(\theta - \pi)$?~~

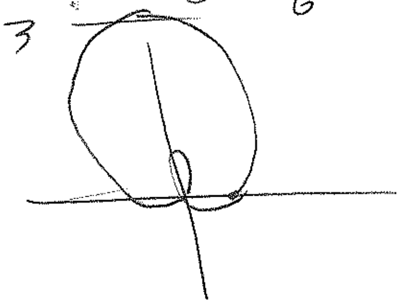
~~$r = 1 - 2\cos\theta$ NO~~

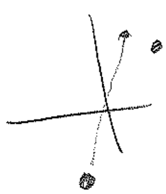
~~$-r = 1 + 2\cos(-\theta)$? NO~~

~~$-r = 1 + 2\cos\theta$~~

~~$r = -1 - 2\cos\theta$~~







$r = 1 + 2\sin(\theta + \pi)$?

$r = 1 - 2\sin\theta$ Nope

~~$r = 1 + \cos$~~

$2\sin\theta + 1 = 0$

$\sin\theta = -\frac{1}{2}$

y-axis again

$r = 1 + 2\sin(\pi - \theta)$

$r = 1 + 2\sin(-(\theta - \pi))$

$= 1 - 2\sin(\theta - \pi)$

$= 1 - 2(-\sin\theta)$

$= 1 + 2\sin\theta$

y-axis Yes

So origin NO

202 §11.4 #s 9, 10, 9, 14, 17, 18, 25, 26 → 13

9 $r^2 = -\sin\theta$

Means $\theta \in [\pi, 2\pi]$ OR $[-\pi, 0]$
but need to keep things below the x-axis

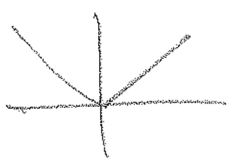
$(r, -\theta)$? No $r^2 = \sin\theta$ is different

$(-r, \pi-\theta)$? $(-r)^2 = r^2 = -\sin(\pi-\theta)$

$= -\sin(-(\theta-\pi))$

$= \sin(\theta-\pi)$

$= -\sin\theta$ $\boxed{\text{Yes x-axis}}$



$(r, \pi-\theta)$?

$r^2 = -\sin(\pi-\theta)$

$= -\sin(-(\theta-\pi))$

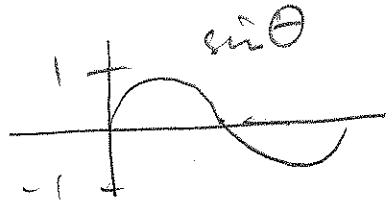
$= \sin(\theta-\pi)$

$= -\sin\theta$ Yes y-axis

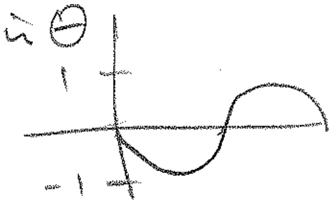
$\boxed{\text{So origin symmetry}}$

$r = \pm\sqrt{-\sin\theta}$ defined only when $-\sin\theta \geq 0$

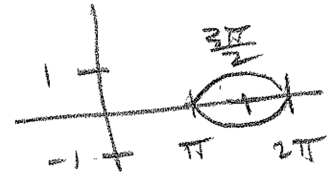
$\sin\theta$

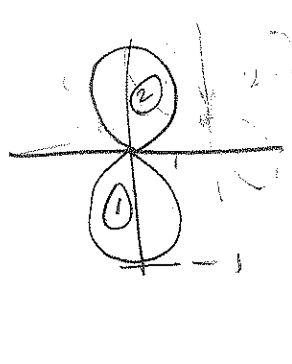


$-\sin\theta$



$\sqrt{-\sin\theta}$



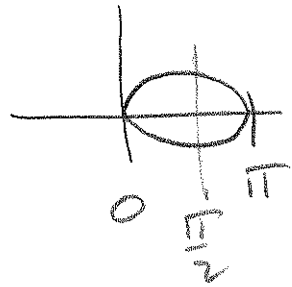


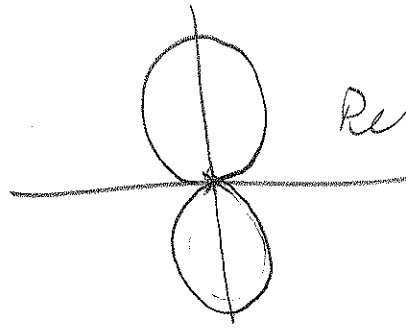
Beanie propeller.
Graphed 1, from π to 2π
Then reflected for 2

207 §11.4 #s 10, 9, 14, 17, 18, 25, 26
→13

10 $r^2 = \sin\theta$ Same exact deal, but need to keep $\theta \in [0, \pi]$

$r = \pm\sqrt{\sin\theta}$





Perfect symmetry!

x - yes

y - yes

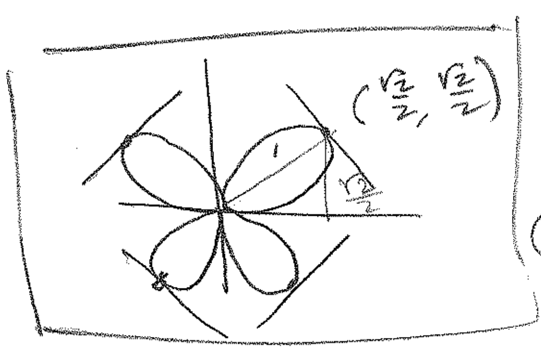
origin - yes

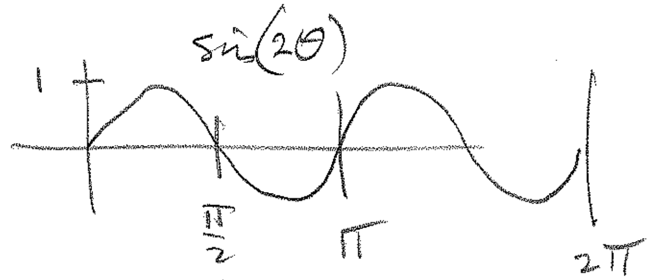
19 #s 17-20

slopes sketch curve and tangent line at the given pts

$r = \sin(2\theta)$ $\theta = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}$

4-leaved rose.





$\left(\pm\frac{\sqrt{2}}{2}, \pm\frac{\sqrt{2}}{2}\right)$ is my gues

$m = \pm 1$ is my guess

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$: $y = -\left(x - \frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2} = -x + \sqrt{2}$

$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ $y = \left(x - \frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2}}{2} = x - \sqrt{2}$

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ $y = \left(x + \frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2} = x + \sqrt{2}$

$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ $y = -\left(x + \frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2}}{2} = -x - \sqrt{2}$

Now lets do the work.

202 §11.4 #s 13, 14, 17, 18, 25, 26

(19) cnt'd - Just realized it was #13, not #19 that was assigned. Finish #19:

$r = \sin(2\theta)$

$\frac{dr}{d\theta} = 2\cos(2\theta)$

(a) $\pm\frac{\pi}{4}, \pm\frac{3\pi}{4}$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{d}{d\theta}[r\sin\theta]}{\frac{d}{d\theta}[r\cos\theta]}$$

$$= \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta} = \frac{2\cos 2\theta\sin\theta + \sin(2\theta)\cos\theta}{2\cos 2\theta\cos\theta - \sin(2\theta)\sin\theta}$$

Let $\theta = \frac{\pi}{4} \Rightarrow$

$$\frac{dy}{dx} = \frac{2\cos(\frac{\pi}{2})\sin\frac{\pi}{4} + \sin\frac{\pi}{2}\cos\frac{\pi}{4}}{2\cos\frac{\pi}{2}\cos\frac{\pi}{4} - \sin\frac{\pi}{2}\sin\frac{\pi}{4}} = \frac{0 + \frac{\sqrt{2}}{2}}{0 - \frac{\sqrt{2}}{2}} = -1$$

Yup.

$\theta = -\frac{\pi}{4}$:

$$\frac{dy}{dx} = \frac{2\cos(-\frac{\pi}{2})\sin(-\frac{\pi}{4}) + \sin(-\frac{\pi}{2})\cos(-\frac{\pi}{4})}{2\cos(-\frac{\pi}{2})\cos(\frac{\pi}{4}) - \sin(-\frac{\pi}{2})\sin(-\frac{\pi}{4})}$$

$$= \frac{0 - \frac{\sqrt{2}}{2}}{0 - \frac{\sqrt{2}}{2}} = +1$$

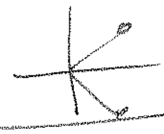
That's enough. picture on previous page is OK

Sort of vague, though.

202 § 11.4 #s 13, 14, 17, 18, 25, 26

~~#~~s 13–16 Check symmetries

(13) $r^2=4\cos(2\theta)$



$(r,-\theta)$: $r^2=4\cos(-2\theta)=4\cos(2\theta)$ Yes

$\boxed{\text{x-axis Yes}}$

$(r,\pi-\theta)$: $r^2=4\cos(+2(\pi-\theta))$

$=4\cos(2(\theta-\pi))$

$=-4\cos(2\theta)$ No

$\boxed{\text{y-axis Yes}}$

$(-r,-\theta)$: $(-r)^2=4\cos(-2\theta)$

$r^2=4\cos(2\theta)$ Yes

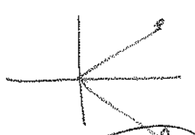
$\boxed{\therefore \text{ origin symmetry}}$

(14) $r^2=4\sin(2\theta)$

Must keep to where $\sin(2\theta)\ge 0$!

$2\theta\in[0,\pi]\cup[2\pi,3\pi]$

$\theta\in[0,\frac{\pi}{2}]\cup[\pi,\frac{3\pi}{2}]$



$r^2=4\sin(-2\theta)$ No

$(-r)^2=4\sin(2(\pi-\theta))$?

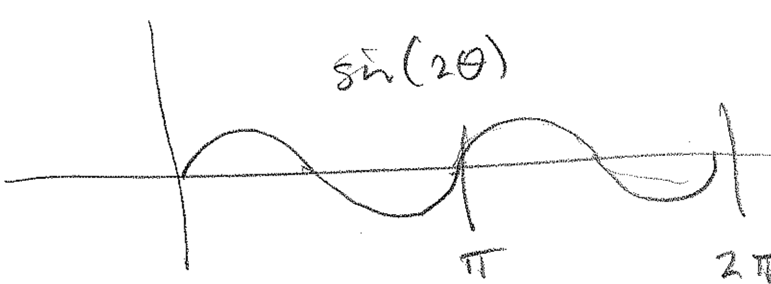
$r^2=4\sin(-2(\theta-\pi))$

$=-4\sin(2(\theta-\pi))$

$=-4\sin(2\theta)$ No

x-axis No

$\sin(2\theta)$

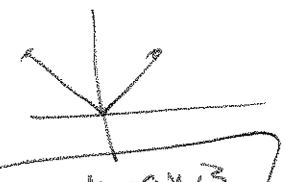


Shift right π ~~Yes~~

Same graph

202 §11.4 #s 14, 17, 18, 25, 26

(14) cnt'd



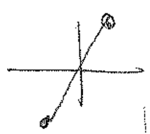
$(r, \pi-\theta)$: $r^2 = 4\sin(2(\pi-\theta))$

$= -4\sin(2(\theta-\pi)) = -4\sin(2\theta)$ No

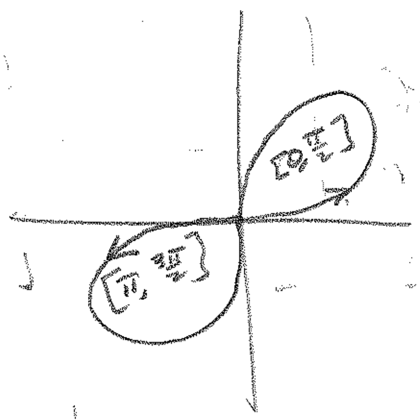
$\boxed{\text{y-axis No } \cancel{\text{Yes}}}$

$(r, \pi+\theta)$ $\quad r^2 = 4\sin(2(\pi+\theta)) = 4\sin(2\pi+2\theta)$

$= 4\sin(2\theta)$ Yes

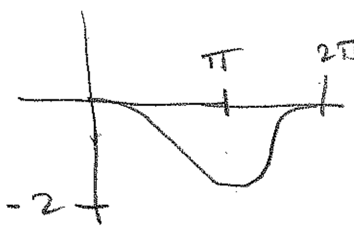


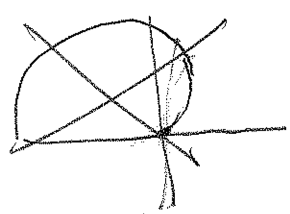
origin

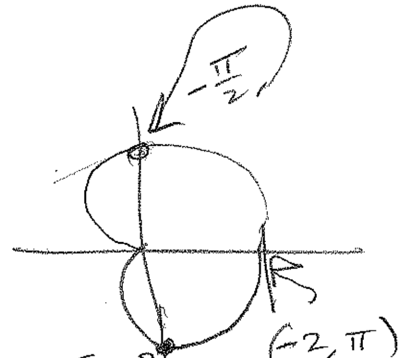


#s 17–20 Sketch curve and tangent(s) @ given pt(s)

(17) $r = -1 + \cos\theta$, $\theta = \pm\frac{\pi}{2}$



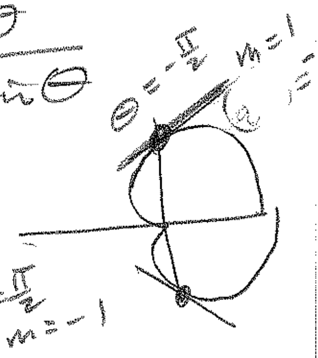




$\frac{dr}{d\theta} = -\sin\theta$

$$\frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta} = \frac{-\sin^2\theta + (\cos\theta - 1)\cos\theta}{-\sin\theta\cos\theta - (\cos\theta - 1)\sin\theta}$$

@ $\frac{\pi}{2}$: $\frac{-1+0}{0+1} = \boxed{-1 = m}$ $\quad$ @ $-\frac{\pi}{2}$: $\frac{-1+0}{0-1} = +1$



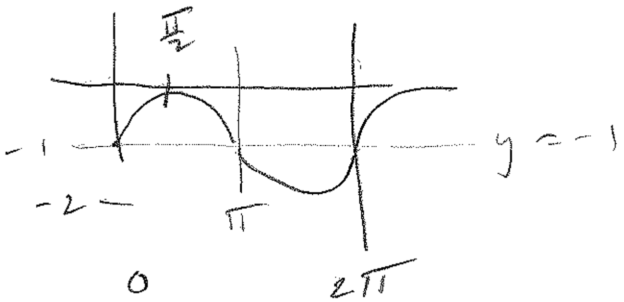
202 §11.4 #s 18, 25, 26

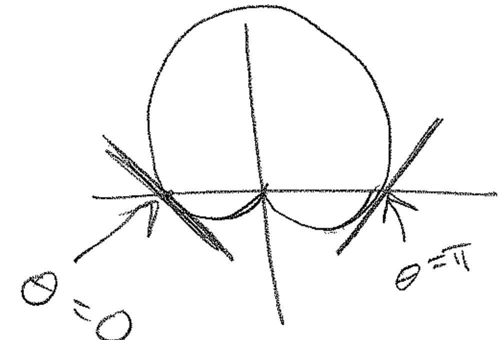
18 $r = -1 + \sin\theta$, $\theta = 0, \pi$ $\qquad r' = \cos\theta$

$$\frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta} = \frac{\sin\theta\cos\theta + (\sin\theta - 1)\cos\theta}{\cos^2\theta - (\sin\theta - 1)\sin\theta}$$

@ $\theta = 0$: $\dfrac{0 - 1}{1} = -1 = m$

@ $\theta = \pi$: $\dfrac{0 + 1}{1 - 0} = \boxed{1 = m}$



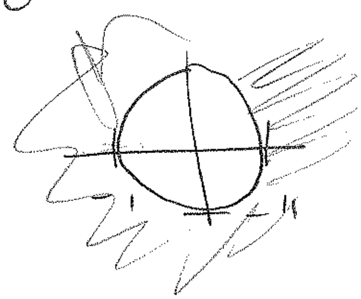


Deceptive, b/c the r-values are all ≤ 1.

#25 Graph: $-1 \le r \le 2\sec\theta$

and $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

$r \le 2\sec\theta \Rightarrow 1 \le \frac{r}{x}$



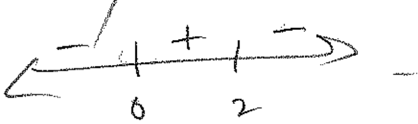
$r \ge -1$

$0 \le x \le 2$

$1 \le \frac{2}{x}$

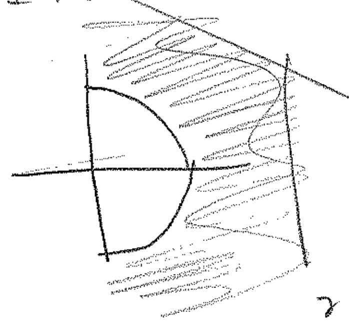
$\frac{2}{x} - 1 \ge 0$

$\frac{2 - x}{x} \ge 0$

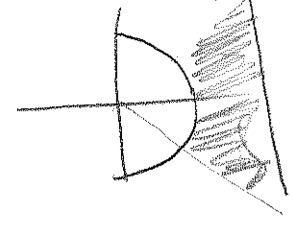


$0 \le x \le 2$

Intersect



Now, $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$



202 S'11 HW #s 25, 26

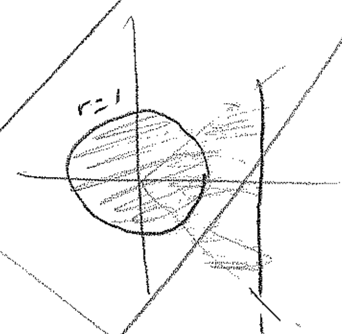
(25) Something's not working

$r = 2\sec\theta = 2\frac{r}{x}$

$1 = \frac{2}{x}$

$x = 2$

Now, θ between $\pm\frac{\pi}{4}$



$r = 1$ OR $r = -1$ is unit circle

$r \le -1$ shade it

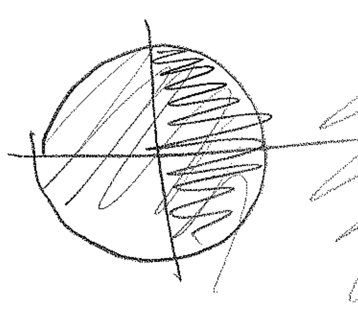
$r \le 2\sec\theta$: shade to left of $x=2$?

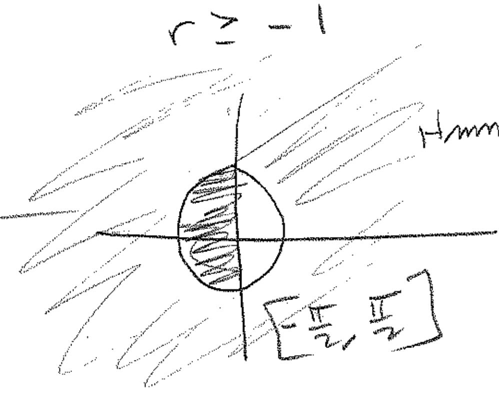
Miscopied the problem!

(25) $-1 \le r \le 2$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$r \le 2$ $\quad$ $r \ge -1$ $\quad$ From $-\frac{\pi}{2}$ to $\frac{\pi}{2}$





$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Hmmmmmmm

I get the two regions, but their intersection is just a line segment.

202 §11.4 #s 25, 26

25 $-1 \le r \le 2$ means

$-1 \le r$ and $r \le 2$

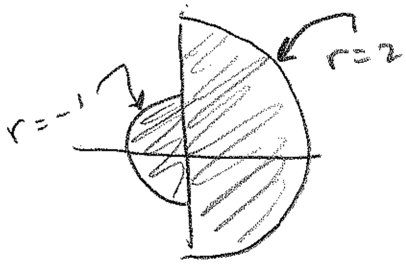
$-1 \le r$

and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

OK. I see it now

$r \ge -1$ means

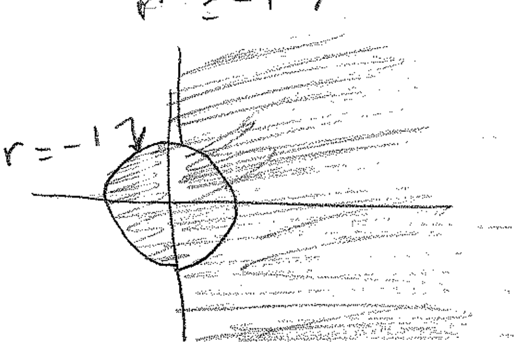
$-1 \le r \le 0$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$



I see these two regions, but I don't see them sharing pts in common and I think that's what's called for.

Try again:

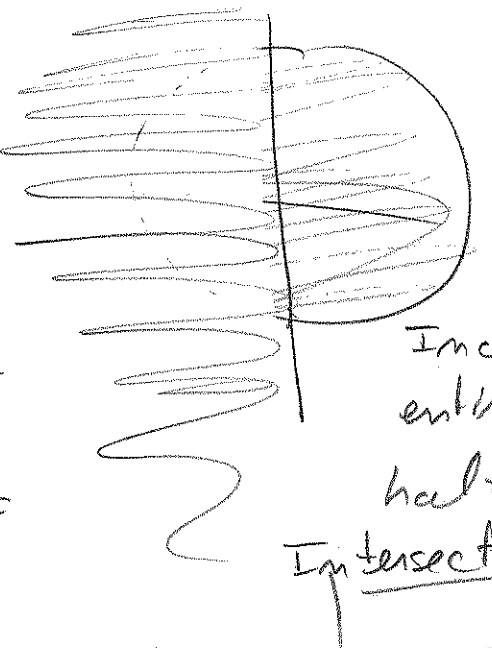
$r \ge -1$:



Includes the right half plane.

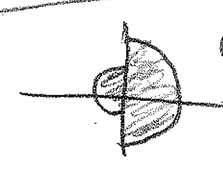
r=.1, r=.7, r=

$r \le 2$



Includes entire **LEFT** half-plane

Intersect the 2:

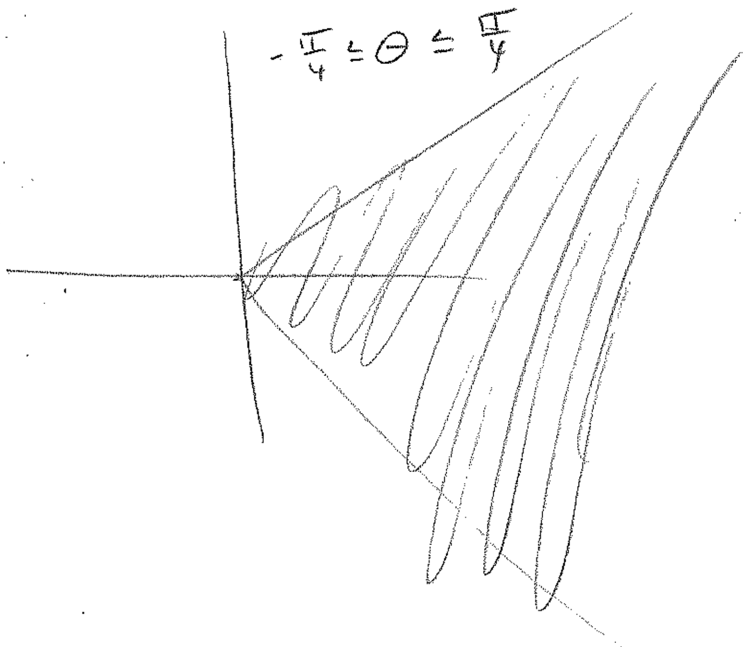


Good

202 S.M.M #26

(26) Now $0 \le r \le 2\sec\theta$

$-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$



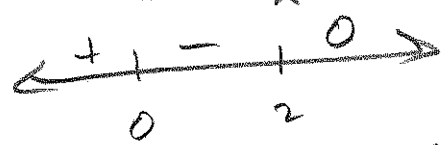
$r \le 2\sec\theta$

$r \le 2\frac{r}{x}$

$1 \le \frac{2}{x}$

If we don't know $x \ge 0$, then $x \le 2$ is illegal,

but $1 - \frac{2}{x} = \frac{x-2}{x} \le 0$ is legit.



Need $x \in (0, 2]$,

Now, is $x = 0$ legit?

I don't think so.

Intersect

I think there's a hole.