202 §10.10 #s 3, 8, 11, 14, 15, 17, 21, 25

#s 1–10 1st 4 terms :

(3) $(1-x)^{-\frac{1}{2}} = (1+(-x))^{-\frac{1}{2}}$

$$= 1 + \sum_{k=1}^{\infty} \binom{-\frac{1}{2}}{k} (-x)^k$$

$$= 1 + \binom{-\frac{1}{2}}{1}(-1)x^1 + \binom{-\frac{1}{2}}{2}(-1)^2x^2 + \binom{-\frac{1}{2}}{3}(-1)^3x^3 + \cdots$$

$$= 1 - \frac{(-\frac{1}{2})}{1!}x + \frac{(-\frac{1}{2})(-\frac{1}{2}-1)}{2!}x^2 - \frac{(-\frac{1}{2})(-\frac{1}{2}-1)(-\frac{1}{2}-2)}{3!}x^3 + \cdots$$

$$= 1 + \frac{1}{2}x + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}x^2 - \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}x^3 + \cdots$$

$$= 1 + \frac{1}{2}x + \frac{\frac{3}{2^2}}{2}x^2 + \frac{\frac{(3)(5)}{2^3}}{3!}x^3 + \cdots$$

$$= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{15}{6 \cdot 8}x^3 + \cdots$$

$$= \boxed{1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3} + \cdots$$

202 §10.10 #s 8, 11, 14, 15, 17, 21, 25

8 $(1+x^2)^{-1/3} = 1 + \sum_{k=1}^{\infty} \binom{-\frac{1}{3}}{k} (x^2)^k$

$= 1 + \binom{-\frac{1}{3}}{1} x^2 + \binom{-\frac{1}{3}}{2} x^4 + \binom{-\frac{1}{3}}{3} x^6 + \cdots$

$= 1 - \frac{1}{3}x^2 + \frac{(-\frac{1}{3})(-\frac{1}{3}-1)}{2!} x^4 + \frac{(-\frac{1}{3})(-\frac{4}{3})(-\frac{7}{3})}{3!} x^6 + \cdots$

$= 1 - \frac{1}{3}x^2 + \frac{(-\frac{1}{3})(-\frac{4}{3})}{2!} x^4 - \frac{\frac{4(7)}{3^3}}{3!} x^6 + \cdots$

$= 1 - \frac{1}{3}x^2 + \frac{4}{3^2 \cdot 2} x^4 - \frac{28}{3^3 \cdot 6} x^6 + \cdots$

$= 1 - \frac{1}{3}x^2 + \frac{2}{9} x^4 - \frac{14}{3^4} x^6 + \cdots$

$= \boxed{1 - \frac{1}{3}x^2 + \frac{2}{9} x^4 - \frac{14}{81} x^6} + \cdots$

202 §10.10 #s 11, 14, 15, 17, 21, 25

#s 11-14 Find the binomial series

(11) $(1+x)^4 = \boxed{1 + 4x + 6x^2 + 4x^3 + x^4}$

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

(14) $\left(1 - \frac{x}{2}\right)^4 = 1 + 4\left(-\frac{x}{2}\right) + 6\left(-\frac{x}{2}\right)^2 + 4\left(-\frac{x}{2}\right)^3 + \left(-\frac{x}{2}\right)^4$

$$= 1 - 2x + \frac{6}{4}x^2 - \frac{4}{8}x^3 + \frac{x^4}{2^4}$$

$$= \boxed{1 - 2x + \frac{3}{2}x^2 - \frac{1}{2}x^3 + \frac{1}{24}x^4}$$

#s 15-18 Estimate the integral w/ a series w/ error < .001

(15) $\displaystyle\int_0^{.2} \sin(x^2)\,dx = \int_0^{.2} \sum_{n=0}^{\infty} (-1)^n \frac{(x^2)^{2n+1}}{(2n+1)!}\,dx$

$$= \int_0^{.2} (-1)^n \frac{x^{4n+2}}{(2n+1)!}\,dx = \left[\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+3}}{(4n+3)(2n+1)!}\right]_0^{.2}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (.2)^{4n+3}}{(4n+3)(2n+1)!}$$

Alternating Series

$|\text{Error}| \le |1^{st} \text{ term in tail}|$

202 §10.10 #s 15, 17, 21, 25

15 contd

There, I had a zero.

$4(0)+3=3$!

$n=0$: $\frac{(.2)^3}{(3)(1)} = \frac{1}{3} \rightarrow .002\overline{6}$

$n=1$: $\frac{(.2)^7}{(7)(3!)} = \frac{.2^7}{42} \approx 3.04761904 \times 10^{-7}$,

So we only need that 1st term!?

$\int_0^{.2} \sin(x^2)\,dx \approx .00266636$, by graphing calculator. This isn't "close" to $\frac{1}{3}$!

$\int_0^{.2} (-1)^n \frac{x^{4n+2}}{(2n+1)!}\,dx = (.2)^3$

OK. Found my mistake. I was up @ the top of the page, the 1st term needs a $(.2)^3$, and I had a $(.2)^0$, there.

Now, it looks Good!

$\frac{(.2)^3}{3} = .002\overline{6}$

Actual $\approx .00266636$

Agrees right up until 7th digit!

202 §10.10 ~~#5~~ ~~#~~21, 25

17. $\int_0^{.1} (1+x^4)^{-\frac{1}{2}} dx$

By previous work:

$$(1-x)^{-\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \cdots$$

so $\left(1-(-x^4)\right)^{-\frac{1}{2}} =$

$$= 1 - \frac{1}{2}(x^4) + \frac{3}{8}(x^4)^2 - \frac{15}{16}(x^4)^3 + \cdots$$

$$= 1 - \frac{1}{2}x^4 + \frac{3}{8}x^8 - \frac{15}{16}x^{12} + \cdots$$

Integrate from 0 to 0.1:

$$\left[x - \frac{1}{2}\cdot\frac{1}{5}x^5 + \frac{3}{8}\cdot\frac{1}{9}x^9 - \frac{15}{16}\cdot\frac{1}{13}x^{13} + \cdots\right]_0^{.1}$$

$$= .1 - \frac{1}{10}(.1)^5 + \frac{3}{72}(.1)^9 - \frac{15}{208}(.1)^{13} + \cdots$$

$\approx .1 - 1\times10^{-6}$ → Already smaller than error tolerance!

$\approx \boxed{.1}$

Graphing Calculator gives $\approx .099999$

Actual error is $\approx .000001 = 10^{-6}$

202 §10.10 #s 21, 25

#s 19-22 Estimate w/i 10^{-8}

21 $\int_0^{0.1} (1+x^4)^{\frac{1}{2}}\,dx$ Thought my eyes tricked me, at first, that I had done #21, already! This is POSITIVE $\frac{1}{2}$ power

$$(1+x)^{\frac{1}{2}} = 1 + \sum_{n=1}^{\infty} \binom{\frac{1}{2}}{n} x^n$$

$$= 1 + \frac{1}{2}x + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3!}x^3 + \cdots$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{3}{8\cdot 3!}x^3 + \cdots$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \cdots$$ SO, integrate:

$$\cancel{\int_0^{.1} [\text{the above}]\,dx = \left[1 + \frac{1}{4}x^2 - \frac{1}{24}x^3 + \frac{1}{48}x^4 + \cdots\right]_0^{.1}}$$

$$\cancel{= 1 + \frac{1}{4}(.1)^2 - \frac{1}{24}(.1)^3 + \frac{1}{48}(.1)^4 -}$$

$\cancel{\frac{2}{10}\cdot\frac{1}{10^4} \approx 10^{-4}/10^{-5}}$ Neup

WAIT! Make the $x^4 \longrightarrow x$ sub:

$$\sqrt{1+x^4} = 1 + \frac{1}{2}x^4 - \frac{1}{8}x^8 + \frac{1}{16}x^{12}$$

$\longrightarrow$ will be smaller than 10^{-8}

202 §10.10 #s 21, 25

21 cont'd

$$\int_0^{.1} \sqrt{1+x^4}\,dx = \int_0^{.1} \left[1 + \frac{1}{2}x^4 - \frac{1}{8}x^8 + \frac{1}{16}x^{12} + \cdots\right] dx$$

$$= \left[x + \frac{1}{2}\cdot\frac{1}{5}x^5 - \frac{1}{8}\cdot\frac{1}{9}x^9 + \frac{1}{16}\cdot\frac{1}{13}x^{13} + \cdots\right]_0^{0.1}$$

I think this will be < error

$$= .1 + \frac{1}{10}(.1)^5 - \frac{1}{72}(.1)^9 + \cdots$$

Yup. $< 10^{-8}$

$$\approx .1 + .1^6 = \boxed{.100001}$$

Calculator: .100001, which is no better than our estimate!

25 #s 25-28 Find a poly that approximates F(x) on [a,b] with error $< 10^{-3}$

202 §10.10 #25

(25) $F(x) = \int_0^x t^2 e^{-t^2}\, dt$ on $[0,1]$

$$e^{-t} = 1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \cdots + (-1)^n \frac{t^n}{n!} + \cdots$$

$$\Rightarrow e^{-t^2} = 1 - t^2 + \frac{t^4}{2!} - \frac{t^6}{3!} + \cdots + (-1)^n \frac{t^{2n}}{n!} + \cdots$$

$$\Rightarrow t^2 e^{-t^2} = t^2 - t^4 + \frac{1}{2!} t^8 + \cdots + (-1)^n \frac{t^{2n+2}}{n!} + \cdots$$

$$\Rightarrow F(x) = \int_0^x \left(\sum_{k=0}^{\infty} (-1)^k \cdot \frac{1}{k!} \cdot t^{2k+2} \right) dx$$

$$= \sum_{k=0}^{\infty} (-1)^k \frac{1}{k!} \cdot \frac{1}{2k+3} \cdot x^{2k+3}$$

$$= \frac{1}{3} x^3 - \frac{1}{1!} \cdot \frac{1}{5} x^5 + \frac{1}{2!} \cdot \frac{1}{7} x^7 - \frac{1}{3!} \cdot \frac{1}{9} x^9 + \cdots$$

on $[0,1]$, we see it's alternating, so error is biggest @ $x=1$;

$\frac{1}{3}, \frac{1}{5}, \frac{1}{14}, \frac{1}{54}, \frac{1}{4!} \cdot \frac{1}{11} = \frac{1}{264}, \frac{1}{5!} \cdot \frac{1}{11}$

$= \frac{1}{120} \cdot \frac{1}{11} = \frac{1}{1320} < \frac{1}{1000} = 10^{-3}$, so

$$\boxed{F(x) \approx \frac{1}{3} x^3 - \frac{1}{5} x^5 + \frac{1}{14} x^7 - \frac{1}{54} x^9 ,}$$

w/in .001 of exact, on $[0,1]$