202 §10.9 #s 7, 16, 4, 22, 33, 38

#s 1–10 Use substitution to find Taylor's series @ $x=0$. (Work off existing power series representations)

(4) $\sin\left(\frac{\pi x}{2}\right) = \left(\frac{\pi x}{2}\right) - \frac{\left(\frac{\pi x}{2}\right)^3}{3!} + \frac{\left(\frac{\pi x}{2}\right)^5}{5!} + \cdots + \frac{(-1)^n \left(\frac{\pi x}{2}\right)^{2n+1}}{(2n+1)!} + \cdots$

$= \frac{\pi}{2}x - \frac{\pi^3}{2^3\, 3!}x^3 + \frac{\pi^5}{2^5\, 5!}x^5 + \cdots + \frac{\pi^{2n+1}(-1)^{2n+1}}{2^{2n+1}(2n+1)!}x^{2n+1} + \cdots$

(7) $\ln(1+x^2)$ Can't find $\ln(1+x)$ *sigh*

$\frac{d}{dx}\left[\ln(1+x)\right] = \frac{1}{1+x} = \sum_{n=0}^{\infty}(-1)^n x^n = 1 - x + x^2 - x^3 + \cdots$

$\Rightarrow \ln(1+x) = C + x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots$

since $\ln(1+0) = 0$, $C = 0$. This gives

$x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots + (-1)^{n+1}\cdot\frac{1}{n}x^n + \cdots$

$= \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}x^n \Longrightarrow$

$$\boxed{\ln(1+x^2) = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}x^{2n}}$$

202 § 10.9 #s 16, 22, 33, 38

#s 11–28 Find Taylor's @ $x=0$.

16 $x^2\cos(x^2)$

$$= x^2\left[1-\frac{(x^2)^2}{2!}+\frac{(x^2)^4}{4!}-\frac{(x^2)^6}{6!}+\cdots+\frac{(-1)^n}{n!}(x^2)^{2n}+\cdots\right]$$

$$= x^2\sum_{k=0}^{\infty}\frac{(-1)^k x^{4n}}{(2n)!} = \boxed{\sum_{k=0}^{\infty}\frac{(-1)^k x^{4n+2}}{(2n)!}}$$

$$= x^2-\frac{x^6}{2!}+\frac{x^{10}}{4!}-\frac{x^{14}}{6!}+\frac{x^{18}}{8!}+\cdots$$

22 $$\frac{2}{(1-x)^3} = 2\left(\frac{1}{(1-x)^3}\right) = f(x)$$

$$\frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{d}{dx}\left((1-x)^{-1}\right) = -\frac{1}{(1-x)^2}$$

$$\frac{d}{dx}\left[-\frac{1}{(1-x)^2}\right] = \frac{d}{dx}\left(-(1-x)^{-2}\right) = 2(1-x)^{-3} = f(x)$$

So, we're looking @ the 2nd derivative wrt x, of $\frac{1}{1-x}$. Nice →

$$\frac{1}{1-x} = 1+x+x^2+x^3+x^4+x^5+\cdots$$

We'd differentiate, twice: →

$$1+2x+3x^2+4x^3+5x^4+\cdots$$

$$2+3\cdot2x+4\cdot3x^2+5\cdot4x^3+\cdots+(n+1)(n)x^{n-1}+\cdots$$

$$= \frac{2}{(1-x)^3} = \sum_{k=1}^{\infty}(k+1)k\cdot x^{k-1}$$

202 S'10.9 #s 33, 38

#s 29-34 1st 4 nonzero terms for maclaurin Series.

(33) $e^{\sin x} = \sum_{k=0}^{\infty} \frac{1}{k!} (\sin x)^k$

$$= \boxed{1 + \sin x + \frac{1}{2!} \sin^2 x + \frac{1}{3!} \sin^3 x + \cdots}$$

(38) Estimate the error for $1 - \frac{x^2}{2}$ on $[-.5, .5]$

Two ways:

Alternating Series:

$$\max_{[-.5,.5]} \left\{ \left| \frac{x^4}{4!} \right| \right\} = \frac{.5^4}{24} \approx \boxed{.0026041\overline{6}}$$

Taylor's Error Estimation:

$$|R_2| \le \frac{\max_{[-.5,.5]} \{ |f^{(3)}(x)| \}}{3!} |x|^3$$

$$\le \frac{\sin(.5)}{6} (.5)^3 \approx \boxed{.009988032|}$$

$f = \cos x$ $\quad f^{(2)} = -\cos x$

$f^{(1)} = -\sin x$ $\quad f^{(3)} = \sin x$

202 § 10.9 #38 cnt'd

38 Note that Taylor's error estimation Theorem gives us a bigger bound on the error. Let's see if I'm getting it right.

$$\cancel{\sin(.5) - \left(1 - \frac{(.5)^2}{2}\right) \approx .3955744614}$$

oops! It'd help if I used cos x!

$$\cos(.5) - \left(1 - \frac{(.5)^2}{2}\right) \approx .0025825619.$$

This falls beneath the error so either answer is correct.

Now, since the NEXT term in this alternating series, with $|a_n|$ decreasing, is positive, I expect $1 - \frac{x^2}{2}$ is an $\boxed{\text{UNDER-estimate}}$, which is confirmed by the positive sign of $\cos(.5) - 1 + \frac{.5^2}{2}$, (but not proved).