202 §10.9 I (Bonus)

① Cosine via Taylor's

② Cosine by $\frac{d}{dx}[\sin x]$

③ $x^2 \cos x$ by ops on #1/2

① $f(x) = \cos x \qquad f(0) = 1$

$f'(x) = -\sin x \qquad f'(0) = 0$

$f^{(2)}(x) = -\cos x \qquad f''(0) = -1$

Repeats —

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \frac{1}{6!}x^6 + \cdots + (-1)^k \frac{x^{2k}}{(2k)!}$$

$$= \boxed{\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}}$$

② $\cos x = \frac{d}{dx}\sin x = \frac{d}{dx}\left[\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}\right]$

$$= \sum_{k=0}^{\infty} (-1)^k \frac{(2k+1)\,x^{2k}}{(2k+1)!} = \boxed{\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}}$$

③ $x^2 \cos x = x^2 \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!} = \boxed{\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+3}}{(2k+1)!}}$