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#s 1–10 $P_0, P_1, \ldots, P_3$ by f @ a

(4) $f(x) = \ln(1+x)$ $a=0$

$f(0) = 0$ $\boxed{P_0 = 0}$

$f'(x) = \frac{1}{1+x}$ $f'(0) = 1$ $\boxed{P_1 = x}$

$f''(x) = -(1+x)^{-2}$ $f''(0) = -\frac{1}{1^2} = -1$

$\boxed{P_2 = x - \frac{1}{2!}x^2}$

STOP!

$\boxed{P_3(x)}$

$f'''(x) = 2(1+x)^{-3}$ $f^{(3)}(0) = 2$

$P_3(x) = x - \frac{1}{2!}x^2 + \frac{2}{3!}x^3 = \boxed{x - \frac{1}{2}x^2 + \frac{1}{3}x^3}$

~~$f^{(4)}(x) = -6(1+x)^{-4}$ $f^{(4)}(0) = \frac{-6}{1}$ $\frac{-6}{4!} = -\frac{1}{4}$~~

~~$\boxed{P_4(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4}$~~

(10) $f(x) = \sqrt{1-x}$ $f(0) = 1$ $\boxed{P_0(x) = 1}$ $(a=0)$

$f'(x) = \frac{1}{2}(1-x)^{-\frac{1}{2}}(-1) \Longrightarrow f'(0) = -\frac{1}{2}$ $\boxed{P_1(x) = -\frac{1}{2}x}$

$f''(x) = -\frac{1}{4}(1-x)^{-\frac{3}{2}} \longrightarrow f''(0) = -\frac{1}{4}$

$P_2(x) = -\frac{1}{2}x + \frac{\frac{1}{4}}{2!}x^2 = \boxed{-\frac{1}{2}x + \frac{1}{8}x^2 = P_2(x)}$

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(10) cont'd

$$f^{(3)}(x) = -\frac{3}{8}(1-x)^{-\frac{5}{2}} \qquad f^{(3)}(0) = -\frac{3}{8}$$

$$\frac{-\frac{3}{8}}{3!} = -\frac{1}{16}$$

$$\boxed{P_3(x) = -\frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3} \qquad \boxed{\text{STOP!}}$$

$$\cancel{f^{(4)}(x) = \frac{15}{16}(1-x)^{-\frac{7}{2}} \qquad f^{(4)}(0) = \frac{15}{16}}$$

$$\cancel{\frac{\frac{15}{16}}{4!} = \frac{15}{16\cdot 4\cdot 3\cdot 2} = \frac{5}{16\cdot 8} = \frac{5}{128}}$$

(14) Find the Taylor's Series @ $x=0$:

(14) $$f(x) = \frac{2+x}{1-x} = \frac{2}{1-x} + \frac{x}{1-x}$$

$$= 2(1+x+x^2+x^3+\dots) + x(1+x+x^2+x^3+\dots)$$

$$= \begin{array}{l} 2+2x+2x^2+2x^3+\dots \quad + \\ \quad\;\; x+x^2+x^3+\dots \end{array}$$

$$2+3x+3x^2+3x^3+\dots$$

$$\boxed{= 2 + \sum_{k=1}^{\infty} 3x^k}$$

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20 $\sinh x = \dfrac{e^x - e^{-x}}{2} =$

$$\tfrac{1}{2}\Big[\Big(1 + x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots\Big) + 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots\Big)\Big]$$

$$= \tfrac{1}{2}\Big[2 + 2\frac{x^2}{2!} + 2\cdot\frac{x^4}{4!} + \cdots\Big]$$

$$= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \boxed{\sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}}$$

Nope. Got the signs wrong on 2nd series. Should subtract.

Subtraction kills even-degree terms &

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$$

$$= \boxed{\sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!} = \sinh x}$$

26 $f(x) = 3x^5 - x^4 + 2x^3 + x^2 - 2$ @ $a = 1$

Interesting : $f(1) = 3 - 1 + 2 + 1 - 2 = 1$

$f'(x) = 15x^4 - 4x^3 + 6x^2 + 2x$

$f'(1) = 15 - 4 + 6 + 2 = 11 + 8 = 19$

$f''(x) = 60x^3 - 12x^2 + 12x + 2$ f

$f''(1) = 60 - 12 + 12 + 2 = 62$

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26 cont'd

$f'''(x) = 180x^2 - 24x + 12 \quad f'''(1) = 180 - 12 = 168$

$f''''(x) = 360x - 24 \quad f^{(4)}(1) = 360 - 24 = 336$

$f^{(5)}(x) = 360 = f^{(5)}(1)$

$f(x) = 1 + 19x + \frac{62}{2!}x^2 + \frac{168}{6}x^3 + \frac{336}{24}x^4 + \frac{360}{120}x^5$

$= 1 + 19x + 31x^2 + 28x^3 + 14x^4 + 3x^5$

__oops!__ Forgot the "x-a" bit:

$1 + 19(x-1) + 31(x-1)^2 + 28(x-1)^3 + 14(x-1)^4 + 3(x-1)^5$

__Perfect!__ Now, do it for $a = -1$, dummy

$f(-1) = -3 - 1 - 2 + 1 - 2 = -7$

$f'(-1) = 15 + 4 + 6 - 2 = 23$

$f''(-1) = -60 - 12 - 12 + 2 = -82$

$f^{(3)}(-1) = 180 + 24 + 12 = 216$

$f^{(4)}(-1) = -360 - 24 = -384$

$f^{(5)}(-1) = 360$

$f(x) = -7 + 23(x+1) - \frac{82}{2!}(x+1)^2 + \frac{216}{3!}(x+1)^3$

$- \frac{384}{4!}(x+1)^4 + \frac{360}{5!}(x+1)^5$

$= \boxed{-7 + 23(x+1) - 41(x+1)^2 + 36(x+1)^3 - 16(x+1)^4 + 3(x+1)^5}$