202 §10.7 II #s 42, 47, 51, 52, 53, 54

#s 41–48 T 20 to find I & S

42 $\sum_{n=0}^{\infty}(e^x-4)^n$ Geometric, in e^x-4.

Converges $\forall$ $|e^x-4|<1 \Longrightarrow$

$-1 < e^x - 4 < 1 \Longrightarrow$

$3 < e^x < 5 \Longrightarrow$

$\{x \mid \ln 3 < x < \ln 5\} = \boxed{I = (\ln 3, \ln 5)}$

for $x \in I$, we have

$$S = \frac{1}{1-(e^x-4)} = \boxed{\frac{1}{5-e^x} = S}$$

47 $\sum_{0}^{\infty}\left(\frac{x^2+1}{3}\right)^n$ Geometric in $\frac{x^2+1}{3} \Longrightarrow$

want $-1 < \frac{x^2+1}{3} < 1$, but, since $\frac{x^2+1}{3} \geq 0$,

this is $0 \leq \frac{x^2+1}{3} < 1 \Longrightarrow$

$0 \leq x^2+1 < 3 \Longrightarrow$

$-1 \leq x^2 < 2 \Longrightarrow$

$x^2 < 2$

$\{x \mid -\sqrt{2} < x < \sqrt{2}\} = I = (-\sqrt{2}, \sqrt{2})$.

$\forall x \in I$, we have $S = \frac{1}{1-\frac{x^2+1}{3}} = \frac{1}{\frac{3-x^2-1}{3}} = \boxed{\frac{3}{2-x^2} = S}$

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51 $\sin x = \sum_0^\infty (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$

(a) 1st 6 terms of cos x :

$1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8 - \frac{1}{10!}x^{10} + \frac{1}{12!}x^{12}$

Should converge ∀ x, since sine does.

b) $\sin(2x) = 2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} \cdots + \frac{(-1)^n (2x)^{2n+1}}{(2n+1)!} + \cdots$

$= 2x - \frac{2^3 x^3}{3!} + \frac{2^5 x^5}{5!} + \cdots + \frac{(-1)^n 2^{2n+1} x^{2n+1}}{(2n+1)!} + \cdots$

converges to sin(2x) ∀ x.

That'll be x^1 thru x^{11}

(c) $2[\sin x \cos x]$ 1st 6 terms

$\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!}\right)\cdot$

$\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!}\right) =$

$x\left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!}\right] +$

$-\frac{x^3}{3!}\left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}\right] +$

$\frac{x^5}{5!}\left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\right] +$

$-\frac{x^7}{7!}\left[1 - \frac{x^2}{2!} + \frac{x^4}{4!}\right] +$

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51 cnt'd

$$+ \frac{x^9}{9!}\left[1 - \frac{x^2}{2!}\right] +$$

$$- \frac{x^{11}}{11!}\left[1\right] \quad =$$

$$x - \frac{1}{2!}x^3 + \frac{1}{4!}x^5 - \frac{1}{6!}x^7 + \frac{1}{8!}x^9 - \frac{1}{10!}x^{11}$$

$$- \frac{1}{3!}x^3 + \frac{1}{2!3!}x^5 - \frac{1}{3!4!}x^7 + \frac{1}{3!6!}x^9 - \frac{1}{3!8!}x^{11}$$

$$+ \frac{1}{5!}x^5 - \frac{1}{5!2!}x^7 + \frac{1}{5!4!}x^9 - \frac{1}{5!6!}x^{11}$$

$$- \frac{1}{7!}x^7 + \frac{1}{7!2!}x^9 - \frac{1}{7!4!}x^{11}$$

$$+ \frac{1}{9!}x^9 - \frac{1}{9!2!}x^{11}$$

$$- \frac{1}{11!}x^{11}$$

$$= x - \left[\frac{1}{2!} + \frac{1}{3!}\right]x^3 +$$

$$= x - \left(\frac{1}{2!} + \frac{1}{3!}\right)x^3 + \left[\frac{1}{4!} + \frac{1}{2!3!} + \frac{1}{5!}\right]x^5$$

LCD = 7!

$$- \left[\frac{1}{6!} + \frac{1}{3!4!} + \frac{1}{5!2!} + \frac{1}{7!}\right]x^7 +$$

$$+ \left[\frac{1}{8!} + \frac{1}{3!6!} + \frac{1}{5!4!} + \frac{1}{7!2!} + \frac{1}{9!}\right]x^9 +$$

$$- \left[\frac{1}{10!} + \frac{1}{3!8!} + \frac{1}{5!6!} + \frac{1}{7!4!} + \frac{1}{9!2!} + \frac{1}{11!}\right]x^{11}$$

$$= x - \left[\frac{3+1}{3!}\right]x^3 + \left[\frac{7+35+21+1}{7!}\right]x^7$$

$$+ \left[\frac{5+10+1}{5!}\right]x^5 + \left[\frac{9+84+126+36+1}{9!}\right]x^9$$

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51 cont'd

$$-\left[\frac{11+165+462+330+55+1}{11!}\right]x^{11}$$

$$=\boxed{x-\frac{4}{3!}x^3+\frac{16}{5!}x^5-\frac{64}{7!}x^7+\frac{256}{9!}x^9-\frac{1024}{11!}x^{11}}$$

= sin x cos x 1st 6 terms

So 2 sin x cos x =

$$2x-\frac{8}{3!}x^3+\frac{32}{5!}x^5-\frac{128}{7!}x^7+\frac{512}{9!}x^9-\frac{2048}{11!}x^{11}$$

= Same as part (b)!

(52) $e^x=\sum_{k=0}^{\infty}\frac{x^k}{k!}=1+x+\frac{x^2}{2}+\frac{x^3}{3!}+\ldots$

(a) $\frac{d}{dx}e^x = 0+1+2\frac{x}{2}+3\frac{x^3}{3!}+\ldots$

$=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots=e^x$

b) $\int e^x dx = x+\frac{1}{2}x^2+\frac{1}{3}\cdot\frac{x^3}{2!}+\frac{1}{4}\cdot\frac{x^4}{3!}+\ldots+C$

Almost the same as e^x, but 1st (constant) term is undetermined. in this work.

(c) We multiply the series for e^x & e^{-x} 1st 6 terms

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52 (c) cnt'd

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \ldots$$

1st 6 terms; stop @ x^5

$$\left(1 - x + \frac{1}{2!}x^2 - \frac{1}{3!}x^3 + \frac{1}{4!}x^4 - \frac{1}{5!}x^5\right)\left(1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5\right)$$

$$= 1\left[1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5\right]$$

$$- x\left[1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4\right]$$

$$+ \frac{1}{2!}x^2\left[1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3\right]$$

$$- \frac{1}{3!}x^3\left[1 + x + \frac{1}{2!}x^2\right]$$

$$+ \frac{1}{4!}x^4\left[1 + x\right]$$

$$- \frac{1}{5!}x^5\left[1\right]$$

$$\begin{aligned}
= 1 + x &+ \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 \\
- x &- x^2 - \frac{1}{2!}x^3 - \frac{1}{3!}x^4 - \frac{1}{4!}x^5 \\
&+ \frac{1}{2!}x^2 + \frac{1}{2!}x^3 + \frac{1}{2!2!}x^4 + \frac{1}{2!3!}x^5 \\
&- \frac{1}{3!}x^3 - \frac{1}{3!}x^4 - \frac{1}{3!2!}x^5 \\
&+ \frac{1}{4!}x^4 + \frac{1}{4!}x^5 \\
&- \frac{1}{5!}x^5
\end{aligned}$$

x^4-term:

$$\frac{2}{4!} + \frac{1}{4} - \frac{2}{3!}$$

$$= \frac{2 + 6 - 8}{4!} = 0 \checkmark \text{ cool}$$

$$1 + 0 + 0 + 0 + 0 + 0 = 1\,!$$

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(53) $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \cdots$

on $(-\frac{\pi}{2}, \frac{\pi}{2})$

(a) 1st 5 terms of $\ln|\sec x|$ are

$\int \tan x\,dx$'s 1st 5 terms:

$$= \frac{1}{2}x^2 + \frac{1}{4\cdot 3}x^4 + \frac{2}{6\cdot 5\cdot 3}x^6 + \frac{17}{8\cdot 315}x^8 + \frac{1}{10}\cdot\frac{62}{2835}x^{10} + \cdots$$

$$= \frac{1}{2}x^2 + \frac{1}{12}x^4 + \frac{2}{90}x^6 + \frac{17}{120}x^8 + \frac{62}{28350}x^{10} + \cdots$$

$$= \boxed{\frac{1}{2}x^2 + \frac{1}{12}x^4 + \frac{1}{45}x^6 + \frac{17}{120}x^8 + \frac{31}{14175}x^{10}}$$

(Ch $\ln\sec 0$| $= 0$ ✓

(b) 1st 5 terms of $\sec^2 x$

$$\frac{d}{dx}[\tan x] = 1 + x^2 + 5\cdot\frac{2}{15}x^4 + 7\cdot\frac{17}{315}x^6 + 9\cdot\frac{62}{2835}x^8 + \cdots$$

$$= 1 + x^2 + \frac{2}{3}x^4 + \frac{17}{45}x^6 + \frac{62}{315}x^8 + \cdots$$

(c) Check by squaring series for $\sec x$

$$\left(1 + \frac{x^2}{2} + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \frac{277}{8064}x^8\right)\left(1 + \frac{x^2}{2} + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \frac{277}{8064}x^8\right)$$

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53 (c) cont'd

$$= 1\left(1 + \frac{x^2}{2} + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \frac{277}{8064}x^8\right)$$

$$+ \frac{x^2}{2}\left(1 + \frac{x^2}{2} + \frac{5}{24}x^4 + \frac{61}{720}x^6\right)$$

$$+ \frac{5}{24}x^4\left(1 + \frac{x^2}{2} + \frac{5}{24}x^4\right)$$

$$+ \frac{61}{720}x^6\left(1 + \frac{x^2}{2}\right)$$

$$+ \frac{277}{8064}x^8(1)$$

24

$2 \lfloor 24$, $2 \lfloor 12$, $2 \lfloor 6$, 3

$2^3 \cdot 3$

$(24)^2 = 2^6 \cdot 3^2$

$2 \lfloor 1440$, $2 \lfloor 720$, $2 \lfloor 360$, $2 \lfloor 180$, $2 \lfloor 90$, $3 \lfloor 45$, $3 \lfloor 15$, 5

$2^5 \cdot 3^2 \cdot 5$

$2 \lfloor 8064$, $2 \lfloor 4032$, $2 \lfloor 2016$, $2 \lfloor 1008$, $2 \lfloor 504$, $2 \lfloor 252$, $2 \lfloor 126$, $3 \lfloor 63$, $3 \lfloor 21$, 7

$2^7 \cdot 3^2 \cdot 7$

$$= 1 + \frac{x^2}{2} + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \frac{277}{8064}x^8$$

$$+ \frac{x^2}{2} + \frac{1}{4}x^4 + \frac{1}{2}\cdot\frac{5}{24}x^6 + \frac{1}{2}\cdot\frac{61}{720}x^8$$

$$\frac{5}{24}x^4 + \frac{5}{24}\cdot\frac{1}{2}x^6 + \frac{5}{24}\cdot\frac{5}{24}x^8$$

$$+ \frac{61}{720}x^6 + \frac{61}{720}\cdot\frac{1}{2}x^8$$

$$+ \frac{277}{8064}x^8$$

$2 \lfloor 40320$, $2 \lfloor 20160$, $2 \lfloor 10080$, $2 \lfloor 5040$, $2 \lfloor 2520$, $2 \lfloor 1260$, $2 \lfloor 630$, $3 \lfloor 315$, $3 \lfloor 105$, $5 \lfloor 35$, 7

$$= 1 + x^2 + \frac{5+6+5}{24}x^4 + \frac{61+75+75+61}{720}x^6$$

$$+ \frac{2\cdot5(277) + 4(7)(61) + 2\cdot5\cdot7\cdot25 + 4(7)61}{40320}$$

$\lfloor 6551$

$$= 1 + x^2 + \frac{2}{3}x^4 + \frac{17}{45}x^6 + \frac{6551}{40320}x^8$$

CAN'T GET THE ARITHMETIC

FOR the x^8-Term.

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(53) (c) Getting that x^8 term:

$$\frac{277}{8064}x^8 + \frac{1}{2}\cdot\frac{61}{720}x^8 + \frac{5}{24}\cdot\frac{5}{24}x^8 + \frac{61}{720}\cdot\frac{1}{2}x^8 + \frac{277}{8064}x^8$$

2 of these

LCD = ?

$8064 = 2^7\cdot 3^2\cdot 7$

$720 = 2^4\cdot 3^2\cdot 5$

$24^2 = 2^6\cdot 3^2$

LCD $= 2^7\cdot 3^2\cdot 5\cdot 7$

But the pair of doubles takes it down to $2^6\cdot 3^2\cdot 5\cdot 7$

$$= \left[2\cdot\frac{277}{8064} + 2\cdot\frac{1}{2}\cdot\frac{61}{720} + \frac{25}{24^2}\right]x^8$$

$$= \frac{277\cdot 5 + 2^2\cdot 7\cdot 61 + 25\cdot 7}{4032}x^8 = \frac{3268}{4032}x^8$$

$2\,|\,3268$
$2\,|\,1634$
$\quad 817$

$= \frac{817}{}$ ok. I give up.

They only wanted 5 terms, anyway.

(54) $\sec x = 1 + \frac{x^2}{2} + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \frac{277}{8064}x^8 + \ldots$

converges for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

converges for $\ln|\sec x + \tan x|$?

(a) 1st 5 terms for

$$\int \sec x\,dx = \int\left(1 + \frac{x^2}{2} + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \frac{277}{8064}x^8\ldots\right)dx$$

$$= C + x + \frac{x^3}{3!} + \frac{5}{5!}x^5 + \frac{61}{7!}x^7 + \frac{277\cdot 5}{9\cdot 8!}x^9$$

$\ln|\sec 0 + \tan 0| = \ln 1 = 0 \longrightarrow C = 0$

$$\frac{277}{72576}x^9 = \frac{1385}{9!}x^9$$

$$= \boxed{x + \frac{1}{3!}x^3 + \frac{1}{24}x^5 + \frac{61}{5040}x^7 + \frac{277}{72576}x^9}$$

$\boxed{\text{Converges for } x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)}$

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54 (b) Find 1st 4 terms of series for

sec x tan x:

$$\sec x \tan x = \frac{d}{dx} \sec x$$

$$= \frac{d}{dx}\left[1 + \frac{x^2}{2} + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \frac{277}{8064}x^8 + \dots\right]$$

$$= \boxed{x + \frac{5}{6}x^3 + \frac{61}{120}x^5 + \frac{277}{1008}x^7}$$

(c) Check by series product: sec x tan x =

$$\left(1 + \frac{x^2}{2} + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \dots\right)\left(x + \frac{x^3}{3} + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \dots\right)$$

$$
\begin{array}{lllll}
= & x + \frac{x^3}{3} & + \frac{2}{15}x^5 & + \frac{17}{315}x^7 \\
+ & \frac{x^3}{2} & + \frac{1}{6}x^5 & + \frac{1}{15}x^7 \\
+ & & \frac{5}{24}x^5 & + \frac{5}{72}x^7 \\
+ & & & \frac{61}{720}x^7 \\
+ & & & \\
\hline
 & x + \frac{5}{6}x^3 & + \frac{61}{120}x^5 & + \frac{277}{1008}x^7
\end{array}
$$

✓