202 §10.7 I #s 4, 15, 22, 29, 30, 35, 36

#s 1-36 (a) Find R & I

(b) Absolute I

Conditional I

(4) $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n}$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(3x-2)^{n+1}}{n+1} \cdot \frac{n}{(3x-2)^n}\right| = \left|\frac{n}{n+1}\right| |3x-2| \xrightarrow{n\to\infty} |3x-2|$$

want $< 1 \Rightarrow |3x-2| < 1 \Rightarrow$

$-1 < 3x-2 < 1 \Rightarrow$

$1 < 3x < 3 \Rightarrow$

$\frac{1}{3} < x < 1$

$x = \frac{1}{3}$:

$$\sum \frac{(3(\frac{1}{3})-2)^n}{n} = \sum \frac{(-1)^n}{n} \text{ converges}$$

$x = 3$:

$$\sum \frac{(3(1)-2)^n}{n} = \sum \frac{1}{n} \text{ diverges}$$

(a) $R = \frac{1-\frac{1}{3}}{2} = \frac{\frac{2}{3}}{2} = \boxed{\frac{1}{3} = R}$

$I = [\frac{1}{3}, 1)$

(b) Cond: $[\frac{1}{3}, 1)$

Abs: $(\frac{1}{3}, 1)$

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15 $\sum_{0}^{\infty} \frac{x^n}{\sqrt{n^2+1}} = \sum a_n$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x^{n+1}}{\sqrt{(n+1)^2+1}} \cdot \frac{\sqrt{n^2+1}}{x^n}\right| = \sqrt{\frac{n^2+1}{(n+1)^2+1}}\,|x| \xrightarrow{n\to\infty} |x|$$

Want $|x|<1$

$$\sqrt{\frac{n^2\left(1+\frac{1}{n^2}\right)}{n^2\left(1+\frac{1}{n}\right)^2+1}} = \sqrt{\frac{\left(1+\frac{1}{n^2}\right)}{\left(1+\frac{1}{n}\right)^2+\frac{1}{n^2}}} \xrightarrow{n\to\infty} 1 \checkmark$$

$x=-1$: $\sum \frac{(-1)^n}{\sqrt{n^2+1}}$ converges

$x=1$: $\sum \frac{1}{\sqrt{n^2+1}}$ diverges (Limit Compare w/ $\sum \frac{1}{n}$

(a) $\boxed{R = 1, \quad I = [-1, 1)}$

(b) Cond: $[-1, 1)$

Abs: $(-1, 1)$

202 §10.7 I #s 22, 29, 30, 35, 36

(22) $\sum_{n=1}^{\infty} \frac{(-1)^n 3^{2n} (x-2)^n}{3n}$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left| \frac{3^{2n+2}(x-2)^{n+1}}{3n+3} \cdot \frac{3n}{3^{2n}(x-2)^n} \right|$$

$$= 3^2 |x-2| \overset{\text{want}}{<} 1 \implies$$

$$|x-2| < \frac{1}{9} \implies$$

$$-\frac{1}{9} < x-2 < \frac{1}{9} \implies$$

$$\frac{17}{9} < x < \frac{19}{9}$$

$$\frac{\frac{19+17}{2}}{9} = \frac{18}{9} =$$

$$3^{2n}\left(\frac{1}{9}\right)^n = (3^2)^n \left(\frac{1}{9}\right)^n = \left(\frac{9}{9}\right)^n \checkmark$$

$x = \frac{17}{9}$: $\sum_{n=1}^{\infty} \frac{(-1)^n 3^{2n}(-\frac{1}{9})^n}{3n} = \sum \frac{3^{2n}(\frac{1}{9})^n}{3n} = \sum \frac{(\frac{9}{9})^n}{3n}$

Diverges

$x = \frac{19}{9}$; Same, but $(-1)^n$'s don't cancel $\implies$

Alternates $\implies$ Converges

(a) $R = \frac{1}{9}$ $\quad I = \left(\frac{17}{9}, \frac{19}{9}\right]$

(b) $\boxed{\begin{array}{l} \text{Cond: } \left(\frac{17}{9}, \frac{19}{9}\right] \\ \text{Abs: } \left(\frac{17}{9}, \frac{19}{9}\right) \end{array}}$

202 §10.7 I #s 29, 30, 35, 36

(29) $\sum_{n=2}^{\infty} \frac{x^n}{n(\ln n)^2}$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left| \frac{x^{n+1}}{(n+1)(\ln(n+1))^2} \cdot \frac{n(\ln(n))^2}{x^n} \right|$$

$$= \frac{n(\ln(n))^2}{(n+1)(\ln(n+1))^2}|x| \xrightarrow{n\to\infty} |x| \overset{\text{want}}{<} 1$$

$\Rightarrow -1 < x < 1$

$x=-1$: Alternates $a_n \to 0$ ✓

Decreasing ✓ Converges

$x=1$: $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$ converges by §10.3 #s 54, 55
($p = 2 > 1$)

(a) $\boxed{R = 1,\ I = [-1, 1]}$

(b) Cnd: $[-1, 1]$

Abs: $[-1, 1]$

202 §10.7 I #s ~~24~~, 30, 35, 36

30 $\sum_{n=2}^{\infty} \frac{x^n}{n \ln n}$ Same deal as #29, but p=1

This will converge @ x = −1, b/c it alternates

Diverges @ x = +1, by 10.3 #s 54, 55, and

p=1 and work done on #29

(a) $R = 1$, $I = [-1, 1)$

(b) $\boxed{\begin{array}{l}\text{Cond}: [-1,1) \\ \text{Abs}: (-1,1)\end{array}}$

35 $\sum_{1}^{\infty} \frac{1+2+3+\cdots+n}{1^2+2^2+3^2+\cdots+n^2} x^n$

$$= \sum_{n=1}^{\infty} \frac{\sum_{k=1}^{n} k}{\sum_{k=1}^{n} k^2} x^n = \sum_{n=1}^{\infty} \frac{\frac{n(n+1)}{2}}{\frac{n(n+1)(2n+1)}{6}} x^n$$

$$= \sum 3 \frac{n(n+1)}{n(n+1)(2n+1)} x^n = \sum \frac{3}{2n+1} x^n$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{3}{2n+3} \cdot \frac{2n+1}{3} x\right| \xrightarrow{n \to \infty} |x| \overset{\text{want}}{<} 1$$

$x = -1$: Alternates, decreases ⇒ converges

$x = +1$: Fails p = 1-test. (limit comparison)

(a) $R = 1$, $I = [-1, 1)$

(b) ~~Ab~~ Cond: $[-1, 1)$

Abs: $(-1, 1)$

202 §10.7 I #s 36

36 $\displaystyle\sum_{n=1}^{\infty} \left(\sqrt{n+1} - \sqrt{n}\right)(x-7)^n$

$$= \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}+\sqrt{n}}(x-3)^n$$

$$\left|\frac{a_{n+1}}{a_n}\right| \xrightarrow{n \to \infty} |x-3| \overset{\text{want}}{<} 1 \longrightarrow$$

$2 < x < 4$

$x = 2$: Alternates, decreases, converges

$x = 4$: Fails $p = \frac{1}{2}$-test, (limit) compare to $\frac{1}{\sqrt{n}}$

(a) $R = 1$, $I = [2, 4)$

(b) Cond.: $[2, 4)$

Abs: $(2, 4)$