202 §10.6 #s 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49
53, 55, 57

#s 1–14 Determine Convergence

(1) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$

Alt ✓

$|a_n| \xrightarrow{n\to\infty} 0$ ✓ $\boxed{\text{converges}}$

(5) $\sum_{n=1}^{\infty} (-1)^{n} \frac{n}{n^2+1}$

Alt ✓

$|a_n| = \frac{n}{n^2+1} \xrightarrow{n\to\infty} 0$ ✓

$\boxed{\text{Converges}}$

(9) $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{n}{10}\right)^n$

Alt ✓

$|a_n| = \left(\frac{n}{10}\right)^n \xrightarrow{n\to\infty} \infty$ $\boxed{\text{Diverges}}$

(13) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}+1}{n+1}$

Alt ✓

$|a_n| = \frac{\sqrt{n}+1}{n+1} = \frac{\sqrt{n}(1+\frac{1}{\sqrt{n}})}{n(1+\frac{1}{n})}$

$= \frac{1}{\sqrt{n}} \left(\frac{1+\frac{1}{\sqrt{n}}}{1+\frac{1}{n}}\right) \xrightarrow{n\to\infty} 0$ $\Rightarrow \boxed{\text{converges}}$

#s 15–48 Absolute/conditional convergence?

Back it up

(17) $\sum_{n=1}^{\infty} (-1)^{n} \frac{1}{\sqrt{n}}$

Converges conditionally
Fails p-test for absolute convergence

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(21) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n+3}$ converges conditionally

Fails p=1-test (limit compare to $b_n = \frac{1}{n}$

(25) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1+n}{n^2}$

$\boxed{\text{Converges conditionally. Fails } p=1\text{-test.}}$

(29) $\sum_{n=1}^{\infty} (-1)^n \frac{\tan^{-1}(n)}{n^2+1}$

$|a_n| = \frac{\tan^{-1}(n)}{n^2+1} \xrightarrow{n\to\infty} \frac{\frac{\pi}{2}}{\infty} = 0$

$\boxed{\text{converges conditionally \& Absolutely.}}$

$\frac{\tan^{-1}(n+1)}{(n+1)^2+1} \cdot \frac{n^2+1}{\tan^{-1}(n)} = \frac{n^2\left(\tan^{-1}(n+1)\right)\left(1+\frac{1}{n^2}\right)}{n^2\left(\left(1+\frac{1}{n}\right)^2+\frac{1}{n^2}\right)\tan^{-1}(n)}$

$= \frac{\tan^{-1}(n+1)\left(1+\frac{1}{n^2}\right)}{\tan^{-1}(n)\left(1+\frac{2}{n^2}+\frac{2}{n^2}\right)} \xrightarrow{n\to\infty} 1$? inconclusive.

Look @ $\frac{\tan^{-1}(n+1)}{\tan^{-1}(n)}$ = Nada

$\frac{\tan^{-1}(n)}{n^2+1} = a_n \leq \frac{\frac{\pi}{2}}{n^2+1} = b_n$, gives us Direct Comparison

202 §10.6 #s 33, 37, 41, 45, 49, 53, 55, 57

(33) $\sum_{n=1}^{\infty} \frac{(-100)^n}{n!} = \sum (-1)^n \frac{100^n}{n!}$

$\boxed{\text{converges absolutely}}$ because $\frac{100^n}{n!} \to 0$ quick

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{100^{n+1}}{(n+1)!} \cdot \frac{n!}{100^n} = \frac{100}{n+1} \xrightarrow{n\to\infty} 0$$

(37) $\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)^n}{(2n)^n}$ Diverges

$$|a_n| = \frac{(n+1)^n}{2^n n^n} = \frac{(n(1+\frac{1}{n}))^n}{2^n n^n} = \frac{n^n (1+\frac{1}{n})^n}{2^n n^n}$$

$$= \frac{(1+\frac{1}{n})^n}{2^n} \xrightarrow{n\to\infty} \frac{e}{\infty} = 0$$

oops! Converges conditionally!

$$\frac{a_{n+1}}{a_n} = \frac{(n+2)^{n+1}}{2^{n+1}(n+1)^{n+1}} \cdot \frac{2^n n^n}{(n+1)^{n+1}}$$

$n+1 = 2\cdot\frac{n}{2} + 2\cdot\frac{1}{2}$

$= 2(\frac{n}{2}+\frac{1}{2})$

$2n+2 = 2(n+1)$

$$= \frac{n^{n+1}(1+\frac{2}{n})^{n+1}}{2 n^{n+1}(1+\frac{1}{n})^{n+1}} \cdot \frac{n^n}{n^{n+1}(1+\frac{1}{n})^{n+1}}$$

$$= \frac{(1+\frac{2}{n})^{n+1}}{2n(1+\frac{1}{n})^{2n+2}} = \frac{\left(1+\frac{1}{(\frac{n}{2})}\right)^{(\frac{n}{2}+\frac{1}{2})(2)}}{n(1+\frac{1}{n})^{(n+1)(2)}} =$$

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CAN be done, eventually, w/ ratio test.

But root test is easier.

$$= \frac{\left(\left(1+\frac{1}{\frac{n}{2}}\right)^{n/2}\left(1+\frac{1}{\frac{n}{2}}\right)^{\frac{1}{2}}\right)^2}{n\left(\left(1+\frac{1}{n}\right)^n\left(1+\frac{1}{n}\right)\right)^2}$$

$$\xrightarrow{n\to\infty} \frac{(e\cdot 1)^2}{n(e\cdot 1)^2} = \frac{1}{n} \xrightarrow{n\to\infty} 0$$

ROOT TEST: $\sqrt[n]{|a_n|} = \sqrt[n]{\frac{(n+1)^4}{(2n)^4}} = \frac{n+1}{2n}$

$\xrightarrow{n\to\infty} \frac{1}{2} <$ $\boxed{\text{converges Absolutely}}$

(41) $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n+1} - \sqrt{n})$ Diverges.

$S_n = -(\sqrt{2} - \sqrt{1}) + (\sqrt{3} - \sqrt{2}) + \dots + (-1)^n(\sqrt{n+1} - \sqrt{n})$

$$\frac{(\sqrt{n+1} - \sqrt{n})(\sqrt{n+1} + \sqrt{n})}{\sqrt{n+1} + \sqrt{n}} = \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}} \xrightarrow{n\to\infty} 0$$

$\Rightarrow$ converges conditionally.

$$S_4 = -(\sqrt{2} - \sqrt{1}) + (\sqrt{3} - \sqrt{2}) - (\sqrt{4} - \sqrt{3}) + (\sqrt{5} - \sqrt{4})$$
$$= -\sqrt{2} + 1 + \sqrt{3} - \sqrt{2} - \sqrt{4} + \sqrt{3} + \sqrt{5} - \sqrt{4}$$
$$= 1 - 2\sqrt{2} + 2\sqrt{3} - 2\sqrt{4} + \sqrt{5}$$
$$=$$

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(41) Try ratio test?

$$\frac{\sqrt{n+2}-\sqrt{n+1}}{\sqrt{n+1}-\sqrt{n}} = \frac{\sqrt{n}\left(\sqrt{1+\frac{2}{n}}-\sqrt{1+\frac{1}{n}}\right)}{\sqrt{n}\left(\sqrt{1+\frac{1}{n}}-1\right)}$$

$$\xrightarrow{n\to\infty} \frac{1-1}{1-1} \text{ indeterminate}$$

$$\xrightarrow{L'H} \frac{\frac{1}{2}\left(1+\frac{2}{n}\right)^{-\frac{1}{2}}\left(-\frac{2}{n^2}\right)-\frac{1}{2}\left(1+\frac{1}{n}\right)^{-\frac{1}{2}}\left(-\frac{1}{n^2}\right)}{\frac{1}{2}\left(1+\frac{1}{n}\right)^{-\frac{1}{2}}\left(-\frac{1}{n^2}\right)}$$

$$= \left[\frac{\frac{1}{\sqrt{1+2/n}}-\frac{1}{2}\cdot\frac{1}{\sqrt{1+1/n}}}{-\frac{1}{2}\left(1+\frac{1}{n}\right)^{-\frac{1}{2}}}\right]$$

CRAP!

Limit comparison to $\frac{1}{\sqrt{n}}$ dummy!

$$a_n = \sqrt{n+1}-\sqrt{n} = \frac{1}{\sqrt{n+1}+\sqrt{n}}$$

$$b_n = \frac{1}{\sqrt{n}}$$

$$\frac{a_n}{b_n} = \frac{\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} = \frac{\sqrt{n}}{\sqrt{n}\left[\sqrt{1+\frac{1}{n}}+1\right]} \xrightarrow{n\to\infty} \frac{1}{2}$$

$\Longrightarrow \sum a_n$ diverges, since $\sum b_n$ does

202 §10.6 #s 45, 49, 53, 55, 57

(45) $\sum_{n=1}^{\infty} (-1)^n \operatorname{sech}(n) = \sum a_n$

$\sinh x = \frac{e^x - e^{-x}}{2}$ $\quad \operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$

$\cosh x = \frac{e^x + e^{-x}}{2}$

Clearly we have conditional convergence.

Now, limit compare to $\sum b_n = \frac{1}{e^x} = e^{-x}$

$$\left|\frac{a_x}{b_x}\right| = \frac{\frac{2}{e^x + e^{-x}}}{\frac{1}{e^x}} = \frac{2e^x}{e^x + e^{-x}} = \frac{e^x[2]}{e^x\left[1 + \frac{1}{e^{2x}}\right]}$$

$\xrightarrow{x \to \infty} 2 \in \mathbb{R} \Rightarrow$ Both converge,

since $\sum e^{-n} = \sum \frac{1}{e^n} = \sum \left(\frac{1}{e}\right)^n$ is

geometric w/ $r = \frac{1}{e} < 1$.

(49) Errors #s 49-52 Estimate error for S_4

(49) $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{1}{n}$

$R_n \le |a_{n+1}| = |a_5| = \boxed{\frac{1}{5}}$

202 § 10.6 #s 53, 55, 57

~~87~~ #s 53-56 How many terms to get $R_n < 0.001$

53 $S = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2+3}$

$|a_1| = \frac{1}{4}$

$|a_2| = \frac{1}{7}$

$|a_3| = \frac{1}{12}$

$|a_4| = \frac{1}{19}$

$|a_5| = \frac{1}{28}$

$|a_6| = \frac{1}{39}$

$|a_7| = \frac{1}{52}$ Take Forever!

Want $\frac{1}{n^2+3} < .001$

$\Rightarrow n^2+3 > 1000$

$\Rightarrow n^2 > 1003$

$\Rightarrow n > \sqrt{1003} \approx 31.67017524$

Use $\boxed{n = 32}$

55 $\sum (-1)^{n+1} \frac{1}{(n+3\sqrt{n})^3}$

Want $\frac{1}{(n+3\sqrt{n})^3} < .001 \Rightarrow$

$(n+3\sqrt{n})^3 > 1000 \Rightarrow$

$n+3\sqrt{n} > \sqrt[3]{1000} = 10$

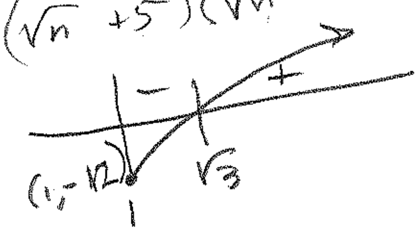
$\Rightarrow n + 3\sqrt{n} - 10 > 0$

$u^2 + 3u - 10 > 0$

$(u+5)(u-3) > 0$

$u = -5$ OR $u = 3 \Rightarrow n = \sqrt{3} \approx 1.$

$(\sqrt{n}+5)(\sqrt{n}-3) > 0$



use $\boxed{n=2.}$

202 § 10.6 #'s 57

57 Want error < 5×10^{-6}

$$\sum_{0}^{\infty} (-1)^n \frac{1}{(2n)!}$$

1 $\frac{1}{2!}$

2 $\frac{1}{4!}$

3 $\frac{1}{6!}$

4 $\frac{1}{8!}$

5 $\frac{1}{10!}$

Want $\frac{1}{(2n)!} < 5 \times 10^{-6}$

$(2n)! > \frac{1}{5} \times 10^{6} = 200,000$

$9! = 362880$

$8! = 40320$

So need $2n \geq 9$, i.e. $\boxed{n \geq 5}$ does it.

$$1 - \frac{1}{2} + \frac{1}{24} - \frac{1}{720} + \frac{1}{40320} - \frac{1}{3628800} \approx .540302303 8$$