202 §10.4 #s 1, 4, 7, 10, 18, 21, 24, 29, 43

#s 1-8 Comparison Test

① $\sum_{n=1}^{\infty} \frac{1}{n^2+30}$ $\quad \frac{1}{n^2+30} > 0$ ✓

$a_n = \frac{1}{n^2+30} < \frac{1}{n^2} = b_n$ & $\sum b_n$ converges

$\Longrightarrow \boxed{\sum a_n \text{ converges}}$

④ $\sum_{n=2}^{\infty} \frac{n+2}{n^2-n}$

$a_n = \frac{n+2}{n^2-n} \leq \frac{n+n}{n^2-\frac{1}{2}n^2} = \frac{2n}{\frac{1}{2}n^2} = \frac{\cancel{4}}{n}$ diverges

So, we want $a_n \geq$ something that diverges

I should've seen divergence right away

(p-test, $n=1$)

$a_n = \frac{n+2}{n^2-n} \geq \frac{n}{n^2} = \frac{1}{n} = b_n$

$\sum b_n$ diverges $\Longrightarrow \boxed{\sum a_n \text{ diverges}}$

⑦ $\sum_{n=1}^{\infty} \sqrt{\frac{n+4}{n^2+4}}$ looks like $a_n \approx \sqrt{\frac{n}{n^2}} = \sqrt{\frac{1}{n}}$

$\sum \sqrt{\frac{1}{n}}$ diverges. so find something smaller

than a_n:

$a_n = \sqrt{\frac{n+4}{n^2+4}} \geq \sqrt{\frac{n}{n^2+\frac{1}{2}n^2}} = \sqrt{\frac{n}{\frac{1}{2}n^2}} = \frac{\sqrt{2}}{\sqrt{n}} = b_n$

$\sum b_n$ diverges $\Longrightarrow \boxed{\sum a_n \text{ diverges}}$

202 §10.4 #10

10 Limit comparison

$$\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^2+2}} = \sum a_n$$

$$\sqrt{\frac{n+1}{n^2+2}} \approx \sqrt{\frac{n}{n^2}} \approx \sqrt{\frac{1}{n}} = \frac{1}{n^{\frac{1}{2}}} \quad \& \quad \sum \frac{1}{n^{1/2}} \text{ diverges}$$

$$\frac{\sqrt{\frac{n+1}{n^2+2}}}{\frac{1}{\sqrt{n}}} = \sqrt{\frac{n+1}{n^2+2}} \cdot \frac{\sqrt{n}}{1} = \sqrt{\frac{n^2+n}{n^2+2}}$$

$$= \frac{n}{n}\sqrt{\frac{1+\frac{1}{n}}{1+\frac{2}{n^2}}} = \sqrt{\frac{1+\frac{1}{n}}{1+\frac{2}{n^2}}} \xrightarrow{n\to\infty} 1 \Rightarrow$$

$\sum a_n$ & $\sum \frac{1}{\sqrt{n}}$ have same convergence props

$\Rightarrow \sum a_n$ diverges

202 §10.4 #s 18, 21, 24, 29, 43

#s 17 – 54 Convergence? Any Method.

(18) $\sum_{1}^{\infty} \frac{3}{n+\sqrt{n}} = \sum a_n$ Direct Comparison NO GOOD

Okey #10 $\frac{3}{n+\sqrt{n}} \le \frac{3}{n}$ & $\sum \frac{3}{n}$ diverges ($p=1$-test)

Limit Comparison to $\sum \frac{1}{n}$

$\frac{3}{n+\sqrt{n}} \cdot \frac{n}{1} = \frac{3}{1+\frac{1}{\sqrt{n}}} \xrightarrow{n \to \infty} 3 \Rightarrow \sum a_n$ diverges b/c $\sum \frac{1}{n}$ diverges

(21) $\sum_{n=1}^{\infty} \frac{2n}{3n-1} = \sum a_n$

$\frac{2n}{3n-1} \ge \frac{2n}{3n} = \frac{2}{3} = b_n$. $\sum b_n$ diverges

$\Rightarrow \boxed{\sum a_n \text{ diverges}}$

(24) $\sum \frac{5n^2-3n}{n^2(n-2)(n^2+5)} = \sum a_n$

$a_n = \frac{5n^2 + \cdots}{n^5 + \cdots} \approx \frac{5}{n^3} = b_n$ & $\sum b_n$ converges

so use limit comparison w/ $\frac{a_n}{b_n}$

$= \frac{n^2(5-\frac{3}{n})}{n^5(1-\frac{2}{n})(1+\frac{5}{n^2})} \cdot \frac{n^3}{5} = \frac{5-3/n}{5(1-\frac{2}{n})(1+\frac{5}{n^2})} \xrightarrow{n \to \infty} 1 \in \mathbb{R}$

$\Rightarrow \boxed{\sum a_n \text{ also converges}}$

202 § 10.4 #s 29, 43

29 $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\ln n}$ I suspect divergesnce.

To prove it, all I need is $\sqrt{n} > \ln n$, eventually. To prove THAT, I need

$\lim_{n\to\infty} \frac{\ln n}{\sqrt{n}} = 0$:

$$\lim_{n\to\infty} \frac{\ln n}{\sqrt{n}} \overset{L'H}{=} \frac{\frac{1}{n}}{\frac{1}{2\sqrt{n}}} = \frac{1}{n}\cdot\frac{2\sqrt{n}}{1} \xrightarrow{n\to\infty} 0$$

So, use limit comparison with $\frac{1}{\sqrt{n}\sqrt{n}} = \frac{1}{n}$, since we know $\sum \frac{1}{n}$ diverges:

$$\frac{a_n}{b_n} = \frac{\frac{1}{\sqrt{n}\ln n}}{\frac{1}{n}} = \frac{n}{\sqrt{n}\ln(n)} = \text{ leads nowhere!}$$

DIRECT comparison with $\frac{1}{n} = \frac{1}{\sqrt{n}\sqrt{n}} \le \frac{1}{\sqrt{n}\ln n}$ $= a_n$. Since $\sum \frac{1}{n}$ diverges and $\frac{1}{n} < \frac{1}{\sqrt{n}\ln n}$ (at least, eventually), $\boxed{\sum \frac{1}{\sqrt{n}\ln n} \text{ diverges}}$

43 $\sum_{n=2}^{\infty} \frac{1}{n!}$ Compare to: $\frac{1}{n^2}$, $\frac{1}{2^n}$, or?

I like 2^n: $\frac{n!}{2^n} = \left[\frac{n\cdot(n-1)\cdots(3)}{2\cdot 2\cdots 2}\right]\left[\frac{(2)(1)}{2\cdot 2}\right] \xrightarrow{n\to\infty} \infty$

so, since $\frac{1}{n!} \le \frac{1}{2^n}$ & $\boxed{\sum \frac{1}{2^n} \text{ converges, so does } \sum \frac{1}{n!}}$