202 §10.3 #s 6, 12, 18, 22, 29, 34, 49, 50, 52

#s 1-10 Use integral test. Check conditions

(6) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$

$f(n) = \dfrac{1}{n(\ln(n))^2} \xrightarrow{n\to\infty} 0$ ✓

$f'(n) = \dfrac{d}{dn}\left[(n(\ln(n))^2)^{-1}\right]$

$= -(n(\ln(n))^2)^{-2}\left[(\ln(n))^2 + n\left(2\ln(n)\left(\frac{1}{n}\right)\right)\right] < 0$ ✓

Integral Test

$$\int_2^{\infty} \frac{dx}{x(\ln x)^2} = \int_2^{\infty} (\ln x)^{-2}\,\frac{1}{x}\,dx$$

$$= \lim_{b\to\infty} \frac{(\ln x)^{-1}}{-1}\Bigg]_2^b = \lim_{b\to\infty}\left[-(\ln b)^{-1} - (-\ln 2)^{-1}\right]$$

$$= 0 + \frac{1}{\ln 2} = \frac{1}{\ln 2} \in \mathbb{R} \Longrightarrow \boxed{\text{Converges}}$$

#s 11-40 Which converge? diverge? Give reasons.

(12) $\displaystyle\sum_{n=1}^{\infty} e^{-n} = \sum_{n=1}^{\infty}\left(\frac{1}{e}\right)^n$ is geometric, with

$r = \dfrac{1}{e} < 1 \Longrightarrow$ Converges

202 §10.3 #s 18, 22, 29, 34, 49, 50, 52

(18) $\sum_{n=1}^{\infty} -\frac{8}{n}$ Diverges, by p-test, $n=1$.

(22) $\sum_{n=1}^{\infty} \frac{5^n}{4^n+3}$ Diverges by n^{th} term test.

$$a_n = \frac{5^n}{4^n+3} = \frac{5^n}{4^n\left(1+\frac{3}{4^n}\right)} = \frac{1}{1+\frac{3}{4^n}}\left(\frac{5}{4}\right)^n \xrightarrow{n\to\infty} \infty$$

(29) $\sum_{n=1}^{\infty} \frac{1}{(\ln 2)^n}$ $\quad a_n = \frac{1}{(\ln 2)^n} = \left(\frac{1}{\ln 2}\right)^n$.

Since $\ln x$ is increasing,

$\ln 2 < \ln e = 1$, from $2 < e$.

So $\frac{1}{\ln 2} > 1$

This is geometric, with $r = \frac{1}{\ln 2} > 1$

$\Rightarrow$ Diverges

(34) $\sum_{n=1}^{\infty} n \tan\left(\frac{1}{n}\right)$

$$\lim_{n\to\infty} n\tan\left(\frac{1}{n}\right) = \lim_{n\to\infty} \frac{\tan\left(\frac{1}{n}\right)}{\frac{1}{n}} \overset{L'H}{=} \lim_{n\to\infty} \frac{\sec^2\left(\frac{1}{n}\right)\left(-\frac{1}{n^2}\right)}{-\frac{1}{n^2}}$$

$$= \lim_{n\to\infty} \sec^2\left(\frac{1}{n}\right) = \sec^2(0) = \frac{1}{\cos^2(0)} = 1 \neq 0$$

Diverges by n^{th} term test.

202 §10.3 #s 49, 50, 52

(49) Estimate $\sum_{n=1}^{\infty} \frac{1}{n^3}$ within 0.01 of actual

$S = S_n + R_n$

$S - S_n = R_n = \sum_{k=n+1}^{\infty} a_n \leq \int_n^{\infty} a_x \, dx$

TERMS ARE POSITIVE & DECREASING.

$= \int_n^{\infty} x^{-3} dx = \lim_{b \to \infty} \left[\frac{x^{-2}}{-2}\right]_n^b = -\frac{1}{2}\left[\lim_{b\to\infty} \frac{1}{b^2} - \frac{1}{n^2}\right]$

$= -\frac{1}{2}\left[-\frac{1}{n^2}\right] = \frac{1}{2n^2}$ want $< 0.01 \Longrightarrow$

$\frac{1}{2(0.01)} < n^2 \Longrightarrow n > \sqrt{\frac{1}{.02}} \approx 7.071067812$

So $n = \cancel{15}$ will do it! $\Rightarrow n = 8$!

Wow!

$1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \cdots + \frac{1}{15^3}$ $\to 8$

Can we do something with an integral instead?

$\int_{n+1}^{\infty} \leq R_n \leq \int_n^{\infty}$ Nah.

$S_n = S - R_n$

~~Nao~~ No guarantees on what happens with the integral.

$1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \frac{1}{5^3} + \frac{1}{6^3} + \frac{1}{7^3} + \frac{1}{8^3}$

$\approx 1.195160244 \approx \boxed{1.195}$

202 §10.3 #s 50, 52

50 want $\sum_{2}^{\infty} \frac{1}{n^2+4}$ within 0.1 of actual

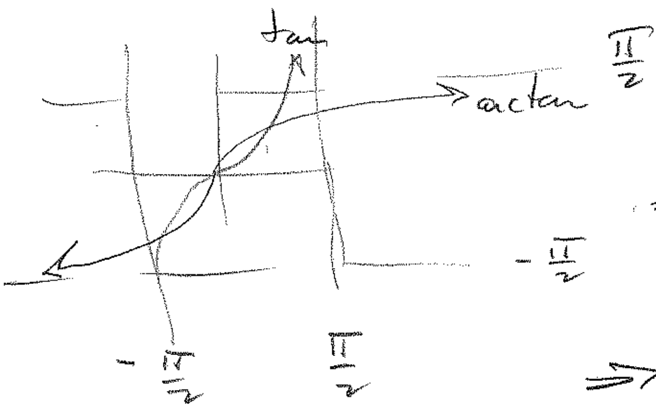
$a_n > 0$ ✓

~~$R_n =$~~ a_n decreasing ✓

$$R_n \le \int_n^\infty \frac{1}{x^2+4}\,dx = \int_n^\infty \frac{dx}{x^2+2^2} = \lim_{b\to\infty} \frac{1}{2}\arctan\left(\frac{x}{2}\right)\Big]_n^b$$

$$= \frac{1}{2}\lim_{b\to\infty}\left[\arctan\left(\frac{b}{2}\right) - \arctan\left(\frac{n}{2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - \arctan\left(\frac{n}{2}\right)\right] = \frac{\pi}{4} - \frac{\arctan\left(\frac{n}{2}\right)}{2} \overset{\text{WANT}}{<} 0.1$$



$$\Rightarrow -\frac{\arctan\left(\frac{n}{2}\right)}{2} < .1 - \frac{\pi}{4}$$

$$\Rightarrow \arctan\left(\frac{n}{2}\right) > -2\left(.1 - \frac{\pi}{4}\right)$$

$$= \frac{\pi}{2} - .2$$

$$\Rightarrow \frac{n}{2} > \tan\left(\frac{\pi}{2} - .2\right)$$

$$\Rightarrow n > 2\tan\left(\frac{\pi}{2} - .2\right) \approx 9.866309751$$

$n = 10$:

$$S_{10} = \frac{1}{2^2+4} + \frac{1}{3^2+4} + \frac{1}{4^2+4} + \cdots + \frac{1}{10^2+4} \approx .5663597329$$

$$\approx \boxed{.57}$$

202 S^4 10.3 #s 52

52 Want $R_n \le 0.01$

for $S^4 = \sum_4^\infty \frac{1}{n(\ln n)^3}$. What's n need 2B?

$a_n \ge 0$ ✓

a_n decreasing ✓

$$\int_n^\infty (\ln n)^{-3} \cdot \frac{1}{n}\,dn = \lim_{b\to\infty} \left.\frac{(\ln n)^{-2}}{-2}\right]_n^b$$

$$= \lim_{b\to\infty}\left[-\frac{1}{2}\left[\frac{1}{(\ln n)^2}\right]_n^b\right]$$

$$= -\frac{1}{2}\left[\lim_{b\to\infty}\frac{1}{(\ln b)^2} - \frac{1}{(\ln n)^2}\right] = \frac{1}{2}\cdot\frac{1}{(\ln n)^2} \quad \text{Want} < 0.01$$

$$\Rightarrow .01 > \frac{1}{2}(\ln n)^{-2}$$

$$\Rightarrow (\ln n)^2 > \frac{1}{.02} = 50$$

$$\Rightarrow \ln n > \sqrt{50} = 5\sqrt{2}$$

$$\Rightarrow n > e^{5\sqrt{2}} \approx 1177.40461$$

$\boxed{n \ge 1178 \text{ does it}}$