202 § 10.2 #s 6, 12, 14, 18, 24, 28, 36, 40, 44

#s 1-6 Find n^{th} partial sum, S_n & use it to find S, if it converges

(6) $\frac{5}{1\cdot 2} + \frac{5}{2\cdot 3} + \frac{5}{3\cdot 4} + \cdots + \frac{5}{n(n+1)} + \cdots$

$$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

$A(n+1) + Bn = 1$

$n=0:\ A=1$

$n=-1:\ -B=1$

$B=-1$

$$\frac{5}{n+1} = 5\left[\frac{1}{n} - \frac{1}{n+1}\right] \Longrightarrow$$

$$S_n = 5\left[\cancel{1} - \cancel{\tfrac{1}{2}} + \cancel{\tfrac{1}{2}} - \cancel{\tfrac{1}{3}} + \cdots + \frac{1}{n} - \frac{1}{n+1}\right]$$

$$= 5\left[1 - \frac{1}{n+1}\right] \xrightarrow{n\to\infty} \boxed{5}$$

#s 7-14 Write out 1st few. Find S

(12) $$\sum_{n=0}^{\infty}\left[\frac{5}{2^n} - \frac{1}{3^n}\right] = 5 - 1 + \frac{5}{2} - \frac{1}{3} + \frac{5}{2^2} - \frac{1}{3^2} + \cdots$$

$$= 5\sum_{0}^{\infty}\frac{1}{2^n} - \sum_{0}^{\infty}\frac{1}{3^n} = \frac{5}{1-\frac{1}{2}} - \frac{1}{1-\frac{1}{3}} = \frac{5}{\frac{1}{2}} - \frac{1}{\frac{2}{3}}$$

$a=5$, $r=\frac{1}{2}$ $\qquad$ $a=1$, $r=\frac{1}{3}$

$$= 10 - \frac{3}{2} = \boxed{\frac{17}{2}}$$

201 §10.2 #s 14, 18, 24, 28, 36, 40, 44

(14) $\sum_{n=0}^{\infty} \frac{2^{n+1}}{5^n} = \sum_{n=0}^{\infty} 2\left(\frac{2}{5}\right)^n = 2 + 2\left(\frac{2}{5}\right) + 2\left(\frac{4}{25}\right) + \ldots$

$= 2\left(\frac{1}{1-\frac{2}{5}}\right) = 2\left(\frac{1}{\frac{3}{5}}\right) = 2\left(\frac{5}{3}\right) = \boxed{\frac{10}{3}}$

#s 15-18 If convergent, find S

(18) $\left(-\frac{2}{3}\right)^2 + \left(-\frac{2}{3}\right)^3 + \ldots + \left(-\frac{2}{3}\right)^{n-2}$

TRICKED US W/ WHERE IT STARTED.

$\cancel{a=1}$, $r = -\frac{2}{3}$

$= \left(-\frac{2}{3}\right)^2\left(-\frac{2}{3}\right)^0 + \left(-\frac{2}{3}\right)^2\left(-\frac{2}{3}\right)^1 + \left(-\frac{2}{3}\right)^2\left(-\frac{2}{3}\right)^2 + \ldots$

$+ \frac{4}{9}\left(-\frac{2}{3}\right)^n + \ldots \qquad a = \frac{4}{9}$

$S = \frac{4}{9}\left(\frac{1}{1-\left(-\frac{2}{3}\right)}\right) = \frac{4}{9}\left(\frac{1}{\frac{5}{3}}\right) = \frac{4}{9}\left(\frac{3}{5}\right) = \boxed{\frac{4}{15}}$

#s 19-26 Express as ratio of 2 integers

(24) $1.\overline{414} = 1 + .414 + .414(.001) + .414(.001)^2 + \ldots + .414(.001)^n$

$= 1 + \sum_{n=0}^{\infty} .414(.001)^n = 1 + \frac{.414}{1-.001}$ = Hmmm m 2 INTEGERS

$= 1 + \frac{.414}{.999} = 1 + \frac{414}{999} = \boxed{\frac{1413}{999}}$

202 § 10.2 #s 28, 36, 40, ...

#s 27–34 TEST FOR DIVERGENCE.

(28) $\sum_{n=1}^{\infty} \frac{n(n+1)}{(n+2)(n+3)} = \sum a_n$

$a_n = \frac{n^2+n}{n^2+5n+6} = \frac{n^2(1+\frac{1}{n})}{n^2(1+\frac{5}{n}+\frac{6}{n^2})} \xrightarrow{n\to\infty} 1 \Longrightarrow$ Diverges.

#s 35–40 If convergent, find sum.

(36) $\sum_{n=1}^{\infty} \left[\frac{3}{n^2} - \frac{3}{(n+1)^2}\right]$ converges, by p-test. (Both converge separately)

$= \frac{3}{1^2} - \frac{3}{2^2} + \frac{3}{2^2} - \frac{3}{3^2} + \cdots + \frac{3}{n^2} - \frac{3}{(n+1)^2}$

$= 3 - \frac{3}{(n+1)^2} \xrightarrow{n\to\infty} \boxed{3}$ (Telescopes)

(40) $\sum_{1}^{\infty} \left(\sqrt{n+4} - \sqrt{n+3}\right) = \sqrt{5} - \sqrt{4} + \sqrt{6} - \sqrt{5} + \sqrt{7} - \sqrt{6}$

$+ \cdots + \sqrt{n+4} - \sqrt{n+3} = -\sqrt{4} + \sqrt{n+1} \xrightarrow{n\to\infty} \infty$

Diverges

Check: $\left(\sqrt{n+4} - \sqrt{n+3}\right)\left(\frac{\sqrt{n+4}+\sqrt{n+3}}{\sqrt{n+4}+\sqrt{n+3}}\right) = \frac{n+4-(n+3)}{\sqrt{n+4}+\sqrt{n+3}}$

$= \frac{1}{\sqrt{n+4}+\sqrt{n+3}} \xrightarrow{n\to\infty} 0$ So the terms do go to zero. Apparently not quickly enough to add 'em all up.

202 §10.2 #44

FIND the sum

$$\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2} = \sum a_n$$

$$\frac{2n+1}{n^2(n+1)^2} = \frac{A}{n} + \frac{B}{n^2} + \frac{C}{n+1} + \frac{D}{(n+1)^2}$$

$$\Rightarrow An(n+1)^2 + B(n+1)^2 + Cn^2(n+1) + Dn^2 = 2n+1$$

$n=0$: $\boxed{B=1}$ $\qquad$ $n=-1$: $D = 2(-1)+1 = \boxed{-1 = D}$

$n=2$:

$$2(3)^2A + 3^2B + 2^2(3)C + 2^2D = 2(2)+1$$

$$18A + 9B + 12C + 4D = 5$$

$$18A + 9 + 12C - 4 = 5$$

$$18A + 12C = 0$$

$$\boxed{9A + 6C = 0}$$

$n=-2$: $(-2)(-1)^2A + B(-1)^2 + C(-2)^2(-1) + D(-2)^2 = -4+1$

$$-2A + B - 4C + 4D = -3$$

$$-2A + 1 - 4C - 4 = -3$$

$$-2A - 4C = 0$$

$$A - 2C = 0$$

$$\boxed{A = 2C}$$

$$9(2C) + 6C = 0$$

$$24C + 0 = \boxed{0 = C} = A$$

$$\frac{1}{n^2} + \frac{-1}{(n+1)^2} = \frac{(n+1)^2 - n^2}{n^2(n+1)^2}$$

$$= \frac{n^2+2n+1-n^2}{(\quad)} = \frac{2n+1}{n^2} \checkmark$$

202 §10.2 # 44

44 So, $a_n = \frac{1}{n^2} - \frac{1}{(n+1)^2}$

$$\Rightarrow \sum a_n = 1 - \frac{1}{2^2} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots + \frac{1}{n^2} - \frac{1}{(n+1)^2} + \cdots$$

$$\Rightarrow S_n = 1 - \frac{1}{(n+1)^2} \xrightarrow{n \to \infty} \boxed{1 = S}$$