202 §8.4

① $\int \frac{dx}{x^2+2x-8}$ $\quad \frac{1}{x^2+2x-8} = \frac{A}{x-2} + \frac{B}{x+4}$

$\Rightarrow 1 = A(x+4) + B(x-2)$

$x=-4 \Rightarrow 1 = -6B \Rightarrow \boxed{B = -\frac{1}{6}}$ $\quad$ If $x=2 \Rightarrow$

$1 = 6A \Rightarrow \boxed{A = +\frac{1}{6}}$

This gives

$$\frac{1}{6}\int \frac{dx}{x-2} - \frac{1}{6}\int \frac{dx}{x+4} = \boxed{\frac{1}{6}\ln|x-2| - \frac{1}{6}\ln|x+4| + C}$$

② $\int \frac{dx}{(x-2)^2(x+4)}$, $\quad \frac{1}{(x-2)^2(x+4)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+4} \Rightarrow$

$1 = A(x-2)(x+4) + B(x+4) + C(x-2)^2$

$x=2 \Rightarrow$

$1 = 6B \Rightarrow \boxed{B = \frac{1}{6}}$

$x=-4 \Rightarrow 1 = 36C$

$\Rightarrow \boxed{C = \frac{1}{36}}$

$\frac{36-43}{36} = -\frac{7}{36}$

$x=3 \Rightarrow$

$1 = A(1)(7) + \frac{1}{6}(7) + \frac{1}{36}(1)$

$1 = 7A + \frac{7}{6} + \frac{1}{36} = 1$

$7A + \frac{43}{36} = 1$

$7A = -\frac{7}{36}$

$\boxed{A = -\frac{1}{36}}$

So

$$\int \frac{dx}{(x-2)^2(x+4)} = -\frac{1}{36}\int \frac{dx}{x-2} + \frac{1}{6}\int \frac{dx}{(x-2)^2} + \frac{1}{36}\int \frac{dx}{x+4}$$

$$= -\frac{1}{36}\ln|x-2| + \frac{1}{6}\cdot\frac{(x-2)^{-1}}{-1} + \frac{1}{36}\ln|x+4| + C$$

$$= \boxed{-\frac{1}{36}\ln|x-2| - \frac{1}{6}\cdot\frac{1}{x-2} + \frac{1}{36}\ln|x+4| + C}$$

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(3) $\int \frac{dx}{x^2+4x+5} = \int \frac{dx}{(x-2)^2+1} = \boxed{\arctan(x-2) + C}$

(4) $\int \frac{dx}{(x-3)(x^2+4x+5)}$ $\frac{1}{(x-3)(x^2+4x+5)} = \frac{A}{x-3} + \frac{Bx+C}{x^2+4x+5}$

$A(x^2+4x+5) + (Bx+C)(x-3) = 1$

$x=3$: $(9-12+5)A = 1$

$2A = 1$

$\boxed{A = \frac{1}{2}}$

$x=4$:

$\frac{1}{2}(16-16+5) + 4B + C = 1$

$\frac{5}{2} + 4B + C = 1$

$4B + C = -\frac{3}{2}$

~~$8B+2C = -3$~~

$C = -4B - \frac{3}{2}$

$x=5$:

$\frac{1}{2}(25-20+5) + (5B+C)(2) = 1$

$5 + 10B + 2C = 1$

$10B + 2C = -4$

$10B + 2(-4B - \frac{3}{2}) = -4$

$2B - 3 = -4$

$2B = -1$

$\boxed{B = -\frac{1}{2}}$

$C = -4(-\frac{1}{2}) - \frac{3}{2}$

$= 2 - \frac{3}{2}$

$= \boxed{\frac{1}{2} = C}$

This gives

$\frac{1}{2}\int \frac{dx}{x-3} + \int \frac{-\frac{1}{2}x + \frac{1}{2}}{x^2+4x+5} dx = \frac{1}{2}\ln|x-3| - \frac{1}{2}\int \frac{x-1}{x^2+4x+5} dx$

$= \frac{1}{2}\ln|x-3| - \frac{1}{2}\cdot\frac{1}{2}\int \frac{2x-2}{x^2+4x+5} dx$

$= \frac{1}{2}\ln|x-3| - \frac{1}{4}\int \frac{2x-4}{x^2+4x+5} dx - \frac{1}{4}\int \frac{2\,dx}{x^2+4x+4}$

$= \boxed{\frac{1}{2}\ln|x-3| + \frac{1}{4}\ln|x^2+4x+5| - \frac{1}{2}\arctan|x-2| + C}$