202 §8.3 #s 6, 10, 17, 22, 23, 36

#s 1–28 Evaluate!

(6) $$\int_0^{\frac{1}{2\sqrt{2}}} \frac{2\,dx}{\sqrt{1-4x^2}} = \int_0^{\frac{1}{2\sqrt{2}}} \frac{2\,dx}{2\sqrt{(\frac{1}{2})^2 - x^2}}$$

Let $x = \frac{1}{2}\sin\theta \Rightarrow 2x = \sin\theta$ & $dx = \frac{1}{2}\cos\theta\, d\theta$

Also, $\theta = \arcsin(2x)$ $\quad -\frac{\pi}{2} \le 2x \le \frac{\pi}{2}$, i.e., $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$

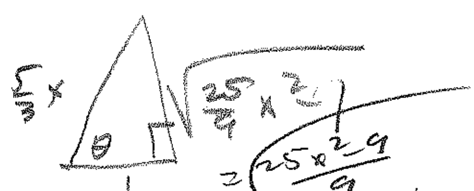
$\arcsin\left(\frac{2}{2\sqrt{2}}\right) = \frac{\pi}{4}$ $\quad \arcsin(0) = 0$. This gives

$$\int_0^{\frac{\pi}{4}} \frac{\frac{1}{2}\cos\theta\, d\theta}{\sqrt{(\frac{1}{2})^2 - (\frac{1}{2})^2\sin^2\theta}} = \int_0^{\frac{\pi}{4}} \frac{\frac{1}{2}\cos\theta\, d\theta}{\frac{1}{2}\sqrt{\cos^2\theta}} \quad (\cos\theta \ge 0)$$

$$= \int_0^{\frac{\pi}{4}} d\theta = \boxed{\frac{\pi}{4}}$$

Students probably avoided making the substitution of the limits of integration.

(10) $$\int \frac{5\,dx}{\sqrt{25x^2-9}} = \int \frac{5\,dx}{5\sqrt{x^2 - (\frac{3}{5})^2}}$$



Let $x = \frac{3}{5}\sec\theta \Rightarrow \frac{5}{3}x = \sec\theta \Rightarrow dx = \frac{3}{5}\sec\theta\tan\theta\, d\theta$

This gives

$$\int \frac{\frac{3}{5}\sec\theta\tan\theta\, d\theta}{\frac{3}{5}\tan\theta} = \int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|\frac{5}{3}x + \frac{\sqrt{25x^2-9}}{3}\right| + C =$$

202 §8.3 #s 17, 22, 23, 26

(17) $$\int \frac{x^3\,dx}{\sqrt{x^2+4}} = \int \frac{8\tan^3\theta \cdot 2\sec^2\theta\,d\theta}{\sqrt{4\tan^2\theta+4}} = \int \frac{16\tan^3\theta\sec^2\theta\,d\theta}{2\sec\theta}$$

Let $x = 2\tan\theta$ $\quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

then $x^3 = 8\tan^3\theta$

& $dx = 2\sec^2\theta\,d\theta$

$$= 8\int \tan^3\theta\sec\theta\,d\theta = 8\int (\sec^2\theta - 1)\sec\theta\tan\theta\,d\theta$$

$$= 8\int (\sec^2\theta\sec\theta\tan\theta - \sec\theta\tan\theta)\,d\theta$$

$$= \boxed{8\ \frac{\sec^3\theta}{3} - \sec\theta + C}$$

(22) $$\int x\sqrt{x^2-4}\,dx = \frac{1}{2}\int (x^2-4)^{\frac{1}{2}}(2x\,dx)$$

$$= \boxed{\frac{2}{3}(x^2-4)^{\frac{3}{2}} + C} = \frac{2}{3}\sqrt{x^2-4}^{\,3} + C$$

(23) $$\int_0^{\frac{\sqrt{3}}{2}} \frac{4x^2\,dx}{(1-x^2)^{3/2}} = \int_0^{\frac{\pi}{3}} \frac{4\sin^2\theta\cos\theta\,d\theta}{\left(\sqrt{\cos^2\theta}\right)^3} =$$

Let $x = \sin\theta$, $dx = \cos\theta\,d\theta$

$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ $\quad 4\int_0^{\frac{\pi}{3}} \frac{\sin^2\theta}{\cos^2\theta}\,d\theta = 4\int_0 (\sec^2\theta - 1)\,d\theta$

$\theta = \sin^{-1}(0) = 0$

$$= \left[4(\tan\theta - \theta)\right]_0^{\frac{\pi}{3}} = 4\left[\sqrt{3} - \frac{\pi}{3} - 0\right] = \boxed{4\sqrt{3} - \frac{4\pi}{3}}$$

202 §8.3 #26

26 $\displaystyle\int_0^{\pi} \sqrt{1-\cos^2\theta}\, d\theta = \int_0^{\pi} |\sin\theta|\, d\theta$

$\displaystyle = \int_0^{\pi} \sin\theta\, d\theta = -\cos\theta\Big]_0^{\pi} = -\cos\pi - (-\cos 0)$

$= 1 + 1 = \boxed{2}$