

202 S8.2 #5 9, 14, 22, 24, 28, 44

$$\textcircled{9} \int \cos^3 x \, dx = \int (1 - \sin^2 x) \cos x \, dx$$

$$= \int (\cos x - \sin^2 x \cos x) \, dx = \boxed{\sin x - \frac{1}{3} \sin^3 x + C}$$

$$\textcircled{14} \int_0^{\frac{\pi}{2}} \sin^2 x \, dx = \int_0^{\frac{\pi}{2}} \frac{1 - \cos(2x)}{2} \, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{2} \, dx - \frac{1}{2} \cdot \frac{1}{2} \int_0^{\frac{\pi}{2}} \cos(2x) \cdot 2 \, dx$$

$$= \left. \frac{1}{2}x \right|_0^{\frac{\pi}{2}} - \left. \frac{1}{4} \sin(2x) \right|_0^{\frac{\pi}{2}} = \frac{1}{2} \cdot \frac{\pi}{2} - \frac{1}{2} \cdot 0 - \left(\frac{1}{4} [\sin \pi - \sin 0] \right)$$

$$= \boxed{\frac{\pi}{4}}$$

$$\textcircled{22} \int_0^{\frac{\pi}{2}} \sin^2(2\theta) \cos^3(2\theta) \, d\theta = \int_0^{\frac{\pi}{2}} \sin^2(2\theta) (1 - \sin^2(2\theta)) \cos(2\theta) \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sin^2(2\theta) \cos(2\theta) \, d\theta - \int_0^{\frac{\pi}{2}} \sin^4(2\theta) \cos(2\theta) \, d\theta$$

$$= \left. \frac{1}{2} \cdot \frac{\sin^3(2\theta)}{3} \right|_0^{\frac{\pi}{2}} - \left. \frac{1}{2} \cdot \frac{\sin^5(2\theta)}{5} \right|_0^{\frac{\pi}{2}} = \boxed{0}$$

202 8.2#s 24, 28, 44

$$\textcircled{24} \int_0^{\pi} \sqrt{1 - \cos(2x)} dx$$

$$= \int_0^{\pi} \sqrt{2\sin^2(x)} dx = \int_0^{\pi} \sqrt{2} \sin x dx =$$

$$= \sqrt{2} [-\cos x]_0^{\pi} = -\sqrt{2} [\cos \pi - \cos 0]$$

$$= -\sqrt{2} [-1 - 1] = \boxed{2\sqrt{2}}$$

$$\textcircled{28} \int_0^{\frac{\pi}{6}} \sqrt{1 + \sin^2 x} dx = \int_0^{\frac{\pi}{6}} \frac{\sqrt{1 - \sin^2 x}}{\sqrt{1 - \sin x}} dx$$

$$= \int_0^{\frac{\pi}{6}} \frac{\cos x dx}{\sqrt{1 - \sin x}} = \int_0^{\frac{\pi}{6}} (1 - \sin x)^{-\frac{1}{2}} \cos x dx$$

$$= - \int_0^{\frac{\pi}{6}} (1 - \sin x)^{-\frac{1}{2}} (-\cos x dx) = - \left. \frac{(1 - \sin x)^{\frac{1}{2}}}{\frac{1}{2}} \right|_0^{\frac{\pi}{6}}$$

$$= -2 \left[(1 - \sin x)^{\frac{1}{2}} \right]_0^{\frac{\pi}{6}} = -2 \left[\left(1 - \frac{1}{2}\right)^{\frac{1}{2}} - (1 - 0)^{\frac{1}{2}} \right] =$$

$$-2 \left[\sqrt{\frac{1}{2}} - 1 \right] = 2 - \frac{2}{\sqrt{2}} = 2 - \frac{2\sqrt{2}}{2} = \boxed{2 - \sqrt{2}}$$

202 § 8.2 #44

$$\textcircled{44} \int \sec^6 x \, dx = \int \frac{1}{\cos^6 x} \, dx$$

$$= \int \frac{1}{(\cos^2 x)^3} \, dx = \int \frac{1}{\left(\frac{1+\cos(2x)}{2}\right)^3} \, dx$$

= ouch! Try:

$$\int (\sec^2 x)^3 \, dx = \int (\tan^2 x + 1)^3 \, dx$$

= Nah. Try to get $\tan^n x \sec^2 x \, dx$

$$\int \sec^6 x \, dx = \int \sec^4 x \sec^2 x \, dx = \int (\tan^2 x + 1)^2 \sec^2 x \, dx$$

$$= \int (\tan^4 x + 2 \tan^2 x + 1) \sec^2 x \, dx$$

$$= \int \tan^4 x \sec^2 x \, dx + 2 \int \tan^2 x \sec^2 x \, dx + \int \sec^2 x \, dx$$

$$= \left[\frac{1}{5} \tan^5 x + \frac{2}{3} \tan^3 x + \tan x + C \right]$$