202 §8.1 #s 26, 32, 40, 54

(26) $\int_0^1 x\sqrt{1-x}\,dx$

Let $u = 1-x \longrightarrow$
$du = -dx$ and $x = 1-u$
$u(0) = 1$, $u(1) = 0$

$$= -\int_1^0 (1-u)\sqrt{u}\,(-du) = \int_0^1 (\sqrt{u} - u\sqrt{u})\,du$$

$$= -\int_0^1 \left(u^{3/2} - u^{1/2}\right) du = -\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right]_0^1 = -\left(\frac{2}{5} - \frac{2}{3}\right)$$

$$= \frac{10-6}{15} = \boxed{\frac{4}{15}}$$

Not sure why this is in the integration by parts, but it's a substitution trick you ought to see ...

(32) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

Let $u = x^{\frac{1}{2}} \longrightarrow$
$du = \frac{1}{2}x^{-\frac{1}{2}}dx = \frac{1}{2\sqrt{x}}\,dx$

This gives
$$2\int (\cos\sqrt{x})\left(\frac{1}{2\sqrt{x}}\,dx\right) = \boxed{-2\sin\sqrt{x} + C}$$

202 § 8.1 #s 40, 54

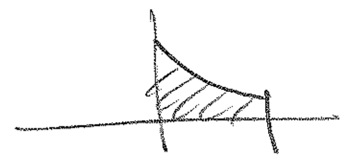
(40) $\int x^2 \sin(x^3)\,dx$

Let $u = x^3 \Longrightarrow$

$du = 3x^2\,dx$

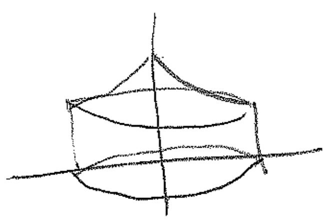
$= \frac{1}{3}\int (\sin(x^3))(3x^2\,dx) = \boxed{-\frac{1}{3}\cos(x^3) + C}$

(54) $y = e^{-x}$, $x = 1$, $y = 0$





(a) revolve about the y-axis



Shell method:

$2\pi \int_0^1 x\, f(x)\,dx = 2\pi \int_0^1 x e^{-x}\,dx$

Let $u = x \rightarrow du = dx$

$dv = e^{-x}\,dx \Longrightarrow$

$v = -e^{-x}$

This gives:

$2\pi \left[ uv - \int_0^1 v\,du \right] = 2\pi \left[ -xe^{-x} \Big|_0^1 + \int_0^1 e^{-x}\,dx \right]$

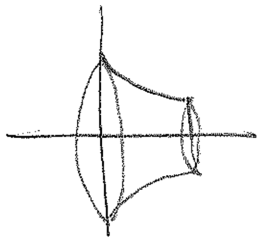
$= 2\pi \left[ -1e^{-1} - 0 - e^{-x} \Big|_0^1 \right] = 2\pi \left[ -\frac{1}{e} - (e^{-1} - e^{0}) \right]$

$= -\frac{2\pi}{e} - \frac{2\pi}{e} + 2\pi = \boxed{\frac{2\pi e - 4\pi}{e} \text{ OR } 2\pi - \frac{4\pi}{e}}$

$\approx 1.6602275908$

202 § 8.1 #54b

54 (b) revolve about x-axis



Disks look good, here.

$$\pi \int_0^1 f(x)^2\,dx = \pi \int_0^1 e^{-2x}\,dx$$

$$= -\frac{\pi}{2} \int_0^1 (e^{-2x})(-2\,dx) = -\frac{\pi}{2}\left[e^{-2x}\right]_0^1$$

$$= -\frac{\pi}{2}\left[e^{-2} - e^{0}\right] = \boxed{-\frac{\pi}{2e^2} + \frac{\pi}{2}} \approx 1.358212161$$