202 S 7.3, 7.4 WS 02

① (a) $y = \ln(5xe^{2x}) \Longrightarrow \boxed{y' = \dfrac{5e^{2x} + (5x)(2e^{2x})}{5xe^{2x}}}$

b) $y = \displaystyle\int_{e^{4\sqrt{x}}}^{e^{2x}} \ln(t)\,dt$

$= \displaystyle\int_0^{e^{2x}} \ln(t)\,dt - \int_0^{e^{4\sqrt{x}}} \ln(t)\,dt \longrightarrow$

$y' = 2\ln(e^{2x}) - \dfrac{4}{2\sqrt{x}} \ln(e^{4\sqrt{x}})$

$= 2\cdot 2x - \dfrac{2}{\sqrt{x}} 4\sqrt{x} = \boxed{4x - 8}$!?

(c) $y = x^{\pi} \longrightarrow \boxed{y' = \pi x^{\pi - 1}}$

(d) $y = \pi^{x} \Longrightarrow \boxed{y' = (\ln \pi)\pi^{x}}$

(e) $y = 2^{\cos(3x)} \Longrightarrow y' = (\ln 2)(2^{\cos(3x)})(-3\sin(3x))$

② (a) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^{\cot(w)} \csc^2(w)\,dw$

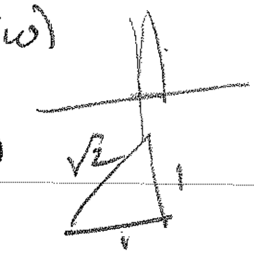
Let $u = \cot(w)$

$\Longrightarrow du = -\csc^2(w)\,dw$

$\Longrightarrow dw = \dfrac{du}{-\csc^2(w)}$

$= \displaystyle\int_1^0 e^{u} \csc^2(w) \frac{du}{-\csc^2(w)}$

$w = \frac{\pi}{2} \Longrightarrow u = \cot\left(\frac{\pi}{2}\right) = 0$



$w = \frac{\pi}{4} \Longrightarrow u = \cot\left(\frac{\pi}{4}\right) = 1$

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(2)(a) cont'd

$$= -\int_1^0 e^u\,du = -\left[e^u\right]_1^0 = -\left[e^0 - e^1\right] = -(1-e) = \boxed{e-1}$$

(2) (b) $\displaystyle\int_0^{\sqrt{\ln\pi}} 2xe^{x^2}\cos(e^{x^2})\,dx$

$u = e^{x^2}$ $\qquad$ $x=0 \Rightarrow u = e^{0^2} = 1$

$du = 2xe^{x^2}dx$ $\qquad$ $x = \sqrt{\ln\pi} \Rightarrow u = e^{(\sqrt{\ln\pi})^2} = e^{\ln\pi} = \pi$

$dx = \dfrac{du}{2xe^{x^2}}$

$$= \int_1^{\pi} 2xe^{x^2}\cos(u)\,\frac{du}{2xe^{x^2}}$$

$$= \int_1^{\pi} \cos u\,du = \sin u\Big]_1^{\pi} = 0 - \sin(1) = \boxed{-\sin 1}\,!?$$

Intuition disagrees!

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2 (c) $\int 7^x\,dx = \frac{1}{\ln 7}\cdot 7^x + C$

$7^x = e^{\ln(7^x)} = e^{x\ln 7} = e^{(\ln 7)x}$

$\xrightarrow{\frac{d}{dx}} \ln 7 \cdot e^{(\ln 7)x}$

$\xrightarrow{\int} \frac{1}{\ln 7} e^{(\ln 7)x} + C$

(d) $\int_1^4 \frac{3^{\sqrt{x}}}{\sqrt{x}}\,dx$

$u = \sqrt{x}$ $\quad x=1 \Rightarrow u=1$

$du = \frac{1}{2\sqrt{x}}\,dx$ $\quad x=4 \Rightarrow u=2$

$dx = 2\sqrt{x}\,du$

$= \int_1^2 \frac{3^u}{\sqrt{x}}\cdot 2\sqrt{x}\,du = 2\int_1^2 3^u\,du$

$= 2\left(\frac{1}{\ln 3}\cdot 3^u\right)\Big|_1^2 = \frac{2}{\ln 3}\left[3^2 - 3^1\right] = \frac{2}{\ln 3}[6]$

$= \boxed{\frac{12}{\ln 3}}$

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(2) (e) $\displaystyle\int_{\frac{1}{10}}^{10} \frac{\log_{10}(10x)}{x}\, dx$

$\log_{10}(10x) = \log_{10}(10) + \log_{10}(x)$

$= 1 + \frac{\ln x}{\ln 10}$

Let $u = \ln x \Rightarrow du = \frac{1}{x}\, dx$

$\Rightarrow dx = x\, du$

$x = \frac{1}{10} \to u = \ln(\frac{1}{10})$

$= \ln 1 - \ln 10 = -\ln 10$

$x = 10 \to u = \ln 10$

$$= \int_{-\ln 10}^{\ln 10} \left(1 + \frac{1}{\ln 10} \cdot u\right) \cdot \frac{1}{x} \cdot x\, du$$

$$= \int_{-\ln 10}^{\ln 10} \left(1 + \frac{1}{\ln 10} \cdot u\right) du$$

$$= \left(u + \frac{1}{\ln 10} \cdot \frac{u^2}{2}\right)\Bigg|_{-\ln 10}^{\ln 10} = \ln 10 + \cancel{\frac{1}{\ln 10} \cdot \frac{(\ln 10)^2}{2}}$$

$$- \left(-\ln 10 + \cancel{\frac{1}{\ln 10} \cdot \frac{(-\ln 10)^2}{2}}\right) = \boxed{2\ln 10 = \ln 100}$$

202 WS #2 cnt'd

3 (a) $\frac{dy}{dt} = e^t \sin(e^t - 2)$, $y(\ln(2)) = 0$

$y = \int e^t \sin(e^t - 2)\, dt + C$

$= \int \sin u \, du + C$

$= -\cos(e^t - 2) + C$

$y(\ln 2) = -\cos(e^{\ln 2} - 2) + C$

$= -\cos(0) + C$

$= -1 + C = 0 \Rightarrow$

$C = 1 \Rightarrow$

$\boxed{y = -\cos(e^t - 2) + 1}$

(b) $y'' = 2e^{-x}$ $y(0) = 1, y'(0) = 0$

$y' = -2e^{-x} + C_1$ $y'(0) = 0 \Rightarrow$

$-2 + C = 0 \Rightarrow C_1 = 2$

$y' = -2e^{-x} + 2 \Rightarrow$

$y = 2e^{-x} + 2x + C_2$

$y(0) = C_2 = 1 \Rightarrow$ $\boxed{y = 2e^{-x} + 2x + 1}$