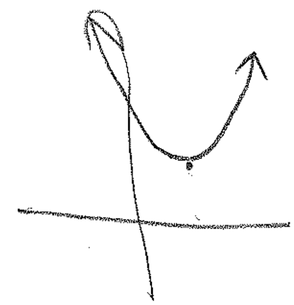
202 §7.1, 7.2 Worksheet

① $f(x) = x^2 - 6x + 13$

$= x^2 - 6x + 3^2 - 9 + 13$

$= (x-3)^2 + 4 \Longrightarrow (h,k) = (3,4)$



Maximal 1-to-1 Domains & Ranges:

$\mathcal{D}_1 = [3, \infty)$; $\mathcal{R}_1 = [4, \infty)$

$\mathcal{D}_2 = (-\infty, 3]$, $\mathcal{R}_2 = [4, \infty)$

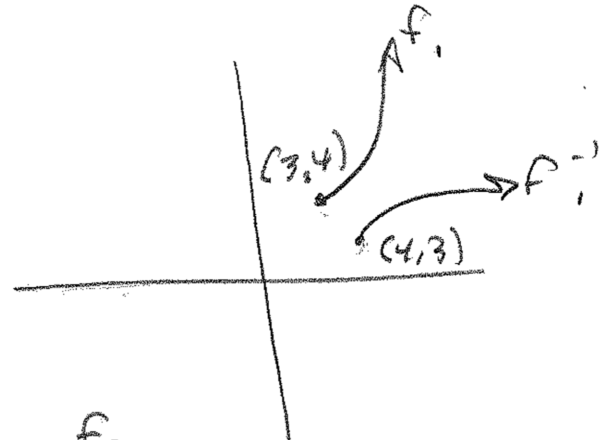
$y = (x-3)^2 + 4$

$x = (y-3)^2 + 4 = x$

$(y-3)^2 = x - 4$

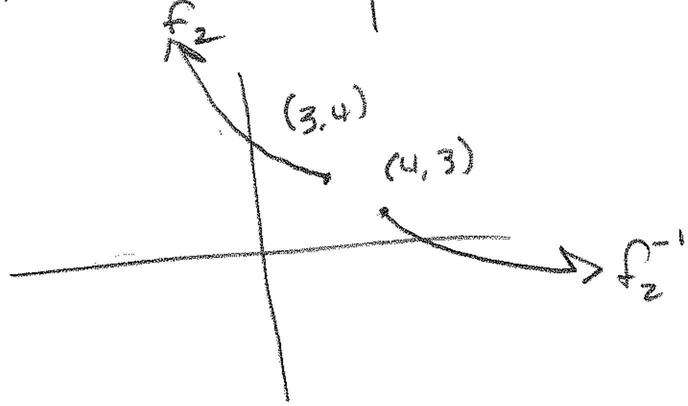
$y - 3 = \pm\sqrt{x-4}$

$y = \pm\sqrt{x-4} + 3$



$f_1^{-1}(x) = \sqrt{x-4} + 3$

$\mathcal{D} = [4, \infty)$, $\mathcal{R} = [3, \infty)$



$f_2^{-1}(x) = -\sqrt{x-4} + 3$

202 §7.1, 7.2

(2) $f(x) = \dfrac{\sqrt{x}}{\sqrt{x}-3}$

$$\frac{\sqrt{y}}{\sqrt{y}-3} = x$$

$$\sqrt{y} = x\sqrt{y} - 3x$$

$$\sqrt{y} - x\sqrt{y} = -3x$$

$$(1-x)\sqrt{y} = -3x$$

$$\sqrt{y} = \frac{-3x}{1-x} = \frac{3x}{x-1}$$

$$\boxed{y = \left(\frac{3x}{x-1}\right)^2 = f^{-1}(x)}$$

(a) $(f^{-1})'(x) = y = \left(\dfrac{3x}{x-1}\right)^2$

$$\Rightarrow \ln y = \ln\left(\left(\frac{3x}{x-1}\right)^2\right) = 2\left(\ln(3x) - \ln(x-1)\right)$$

$$\Rightarrow \frac{y'}{y} = 2\left(\frac{3}{3x} - \frac{1}{x-1}\right) \longrightarrow$$

$$y' = 2\left(\frac{3x-3-3x}{3x(x-1)}\right)\left(\frac{3x}{x-1}\right)^2$$

$$= \frac{-6(9x^2)}{3x(x-1)^3} = \frac{-54x^2}{3x(x-1)^3} = (f^{-1})'(x) \Rightarrow$$

$$(f^{-1})'(2) = \frac{-54(2^2)}{3(2)(2-1)^3} = \frac{-54(4)}{6} = -9(4) = \boxed{-36}$$

202 §7.1, 7.2

2 (b) $f(x) = y = \dfrac{\sqrt{x}}{\sqrt{x}-3} = \dfrac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}-3} \overset{\text{SET}}{=} 2$

$$\Longrightarrow \sqrt{x} = 2\sqrt{x} - 6$$

$$\sqrt{x} - 2\sqrt{x} = -6$$

$$-\sqrt{x} = -6$$

$$\sqrt{x} = 6$$

$\boxed{x = 36} \Longrightarrow \boxed{(a,b) = (36, 2)}$ and $f^{-1}(b) = \boxed{f^{-1}(2) = 36}$

$$\ln y = \ln\left(\frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}-3}\right) = \ln\left(x^{\frac{1}{2}}\right) - \ln\left(x^{\frac{1}{2}}-3\right) \Longrightarrow$$

$$= \frac{1}{2}\ln(x) - \ln\left(x^{\frac{1}{2}}-3\right)$$

$$\frac{y'}{y} = \frac{1}{2}\cdot\frac{1}{x} - \frac{\frac{1}{2}x^{-\frac{1}{2}}}{x^{\frac{1}{2}}-3} = \frac{1}{2x} - \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}-3} \Longrightarrow$$

$$y' = \left(\frac{1}{2x} - \frac{1}{2\sqrt{x}(\sqrt{x}-3)}\right)\left(\frac{\sqrt{x}}{\sqrt{x}-3}\right) = f'(x) \Longrightarrow$$

$$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(36)} =$$

$$= \frac{1}{\left(\frac{1}{2(36)} - \frac{1}{2\sqrt{36}(\sqrt{36}-3)}\right)\left(\frac{\sqrt{36}}{\sqrt{36}-3}\right)} = \frac{1}{\left(\frac{1}{72} - \frac{1}{2(6)(6-3)}\right)\left(\frac{6}{6-3}\right)}$$

$$= \frac{1}{\left(\frac{1}{72} - \frac{1}{36}\right)(2)} = \frac{1}{\left(-\frac{1}{72}\right)(2)} = \boxed{-36}$$

202 §7.1, 7.2

3. $f(x) = x^3 - 4x^2 - 7x + 13 \overset{\text{SET}}{=} 3 \Rightarrow$

$x^3 - 4x^2 - 7x + 10 = 0$

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -7 & 10 \\ & & 1 & -3 & -10 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

So, $x = 1 \in [-1, 3]$ is a root – The ONLY root in there, if my 1-to-1 claim on $[-1, 3]$ be true.

So, $(a, b) = (1, 3) = (a, f(a)) = (f^{-1}(b), b)$

$f'(x) = 3x^2 - 8x - 7$

$f'(f^{-1}(b)) = f'(1) = 3 - 8 - 7 = -12$

$$\Rightarrow (f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(1)} = \boxed{\frac{1}{-12}}$$

4. $\ln(35) + \dfrac{\ln\left(\frac{1}{7}\right)}{\ln(25)} =$

$$= \ln 5 + \ln 7 + \frac{\ln(1) - \ln(7)}{2\ln 5} = \boxed{\ln 5 + \ln 7 - \frac{\ln 7}{2\ln 5}}$$

202 §7.1, 7.2

5 (a) $\frac{d}{dx}\left[\ln(\sec x + \tan x)\right]$

$$= \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \frac{\sec x\,(\tan x + \sec x)}{\sec x + \tan x} = \boxed{\sec x}$$

$$\Longrightarrow \boxed{\int \sec x\,dx = \ln(\sec x + \tan x) + C}$$

(b) $\frac{d}{dx}\left[\ln(x^3)\right] = \frac{d}{dx}\left[3\ln x\right] = \boxed{\frac{3}{x}}$

(c) $\frac{d}{dx}\left[(\ln(x))^3\right] = \boxed{3(\ln(x))^2\left(\frac{1}{x}\right)}$

(d) $\frac{d}{dx}\left[\ln\left(\frac{x^3-17x}{5x^2+27}\right)\right] = \frac{d}{dx}\left[\ln(x^3-17x) - \ln(5x^2+27)\right]$

$$= \boxed{\frac{3x^2-17}{x^3-17x} - \frac{10x}{5x^2+27}}$$

202 §7.1, 7.2

6 (a) $\frac{d}{d\theta}\left[\sqrt{\frac{\theta}{\theta+1}}\right] = ?$

$$y = \sqrt{\frac{\theta}{\theta+1}} \Longrightarrow$$

$$\ln y = \ln\left(\left(\frac{\theta}{\theta+1}\right)^{\frac{1}{2}}\right) = \frac{1}{2}\left[\ln(\theta) - \ln(\theta+1)\right] \Longrightarrow$$

$$\frac{y'}{y} = \frac{1}{2}\left[\frac{1}{\theta} - \frac{1}{\theta+1}\right] \Longrightarrow$$

$$\boxed{y' = \frac{1}{2}\left[\frac{1}{\theta} - \frac{1}{\theta+1}\right]\left(\sqrt{\frac{\theta}{\theta+1}}\right)}$$

(b) $y = \sqrt[7]{\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)}} \Longrightarrow$

$$\ln y = \frac{1}{7}\left[\ln x + \ln(x+1) + \ln(x-2) - \ln(x^2+1) - \ln(2x+3)\right]$$

$$\Longrightarrow \frac{y'}{y} = \frac{1}{7}\left[\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x-2} - \frac{2x}{x^2+1} - \frac{2}{2x+3}\right] \Longrightarrow$$

$$\boxed{y' = \frac{1}{7}\left[\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x-2} - \frac{2x}{x^2+1} - \frac{2}{2x+3}\right]\sqrt[7]{\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)}}}$$