$$\lim_{n\to\infty} 3^{\frac{1}{n}} = 3^0 = 1$$

$$\left.\begin{array}{l} 3^x \text{ increasing} \\ \frac{1}{n} \text{ decreasing} \end{array}\right\} \Longrightarrow 3^{\frac{1}{n}} \text{ decreasing}$$

$$3^{\frac{1}{n}} > 0 \quad \forall n \in \mathbb{N} \Longrightarrow \text{Bdd below}$$

$\Longrightarrow$ Converges.

$$\frac{d}{dn}\left[3^{\frac{1}{n}}\right] = (\ln 3)(3^{\frac{1}{n}})\left(-\frac{1}{n^2}\right) < 0 \quad \forall n \in \mathbb{N}$$

$\longrightarrow$ decreasing.

$$\begin{array}{rrl} - & 3.\overline{25} & = x \\ + & 325.\overline{25} & = 100x \\ \hline & 322 & = 99x \end{array}$$

$$\frac{322}{99} = x$$

$$3 + \frac{25}{100} + \frac{25}{100^2} + \frac{25}{100^3} + \cdots$$

$$= 3 + \frac{25}{100}\left[1 + \frac{1}{100} + \frac{1}{100^2} + \cdots\right]$$

$$= 3 + \frac{25}{100}\left[\sum_{n=1}^{\infty}\left(\frac{1}{100}\right)^{n-1}\right]$$

$$= 3 + \frac{25}{100}\left[\frac{1}{1-\frac{1}{100}}\right]$$

$$= 3 + \frac{25}{100}\left[\frac{1}{\frac{99}{100}}\right] =$$

$$= 3 + \frac{25}{100}\left[\frac{100}{99}\right]$$

$$= 3 + \frac{25}{99}$$

$$= \frac{297 + 25}{99} = \frac{322}{99}$$

(4) $S_5 = \sum_{k=1}^{5}$ doesn't make sense.

Should've had $S_5 = \sum_{k=2}^{5} \frac{1}{k(\ln k)^4}$

$$\text{Error} = R_5 = \sum_{k=6}^{\infty} \frac{1}{k(\ln k)^4} \le \int_5^{\infty} \frac{dx}{x(\ln x)^4} = I$$

$$\left(\begin{array}{l} u = \ln x \quad du = \frac{1}{x}dx \\ u(5) = \ln 5 \\ \text{"} u(\infty) = \ln(\infty) = \infty \text{"} \end{array} \right)$$

$$\Rightarrow I = \int_{\ln 5}^{\infty} u^{-4}\,du = \lim_{t\to\infty} \int_{\ln 5}^{t} u^{-4}\,du =$$

$$\lim_{t\to\infty} \left[-\frac{1}{3}u^{-3}\right]_{\ln 5}^{t} = \lim_{t\to\infty} -\frac{1}{3}\cdot\frac{1}{t^3} - \left(-\frac{1}{3}\frac{1}{(\ln 5)^3}\right)$$

$$= \frac{1}{3(\ln 5)^3} \ge R_5$$

---

want $n \ni R_n \le .01$

want $\int_n^{\infty} \frac{dx}{x(\ln x)^4} < .01$, i.e., want

$$\frac{1}{3(\ln(n))^3} < .01 = \frac{1}{100}$$

$$100 < 3(\ln(n))^3$$

$$3(\ln(n))^3 > 100$$

$$(\ln(n))^3 > \frac{100}{3}$$

$$\ln(n) > \sqrt[3]{\frac{100}{3}}$$

$$n > e^{\sqrt[3]{\frac{100}{3}}} \approx 24.99. \text{ Use } n = 25$$

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{(n+3\sqrt{n})^3} \qquad \text{Want error} < .001$$

$\frac{1}{(n+3\sqrt{n})^3}$ is strictly decreasing.
(n inc., $\sqrt{n}$ inc., $n+3\sqrt{n}$ inc.)

Want $\boxed{\frac{1}{(n+3\sqrt{n})^3} < .001} = \frac{1}{1000}$

$(n+3\sqrt{n})^3 > 1000$

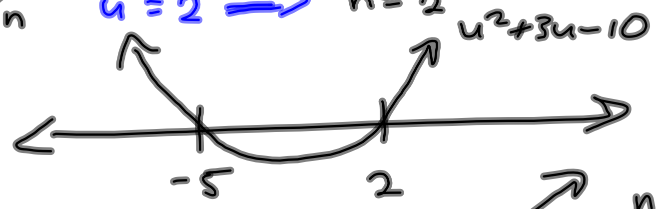
$n + 3\sqrt{n} > 1000^{\frac{1}{3}} = 10$

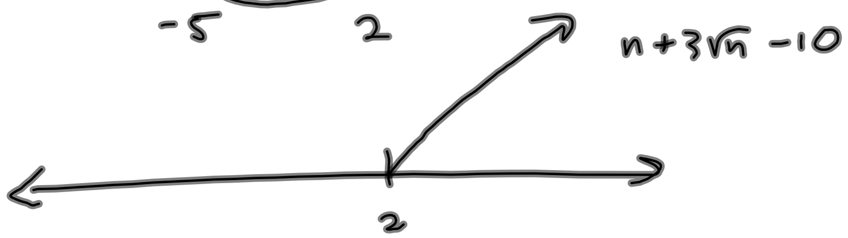
$u = \sqrt{n}$ $\qquad n + 3\sqrt{n} - 10 > 0$

$u^2 + 3u - 10 > 0$

$(u+5)(u-2) > 0$

$u = -5 = \sqrt{n}$ $\qquad u = 2 \Longrightarrow n = 2^2$





$\boxed{\#5_c}$ Wanted to ask

$$\sum_{n=500}^{\infty} \frac{n^2-3n-2}{n^3+n^2+11}$$ for Direct Comparison (Tough!)

$$a_n = \frac{n^2-3n-2}{n^3+n^2+11} \geq \frac{n^2-\frac{1}{2}n^2-\frac{1}{4}n^2}{n^3+\frac{1}{2}n^3+\frac{1}{4}n^3} = \frac{\frac{1}{4}n^2}{\frac{7}{4}n^3} = \frac{1}{4}\cdot\frac{4}{7}\cdot\frac{1}{n}$$

a_n (eventually) > 0

$$= \frac{1}{7}\cdot\frac{1}{n} = b_n$$

a_n eventually decreasing?

$\sum b_n$ diverges.

$$\frac{(2n-3)(n^3+n^2+11)-(n^2-3n-2)(3n^2+2n)}{(\quad)^2}$$ Done.

$$= \frac{2n^4 + \text{smaller} - 3n^4 + \text{smaller}}{(\quad)^2}$$ eventually negative. ✓

Limit comparison

$\lim_{n\to\infty} \left| \frac{a_n}{b_n} \right| = c \neq 0$ Both share same props.

Σ

To show convergence: S Σb_n converges.

$\boxed{\lim_{n\to\infty} \left| \frac{a_n}{b_n} \right| = 0} \Rightarrow \Sigma a_n$ converges.

Σ

$\lim_{n\to\infty} \left| \frac{a_n}{b_n} \right| = \infty \Rightarrow$ nothing Σa_n

$\left| \frac{\frac{1}{n^2}}{\frac{1}{n^3}} \right| \xrightarrow{n\to\infty} \infty$

Σa_n

S Σb_n diverges

$\left| \frac{a_n}{b_n} \right| \xrightarrow{n\to\infty} 0 \Rightarrow$ Nothing Σa_n

$\left| \frac{a_n}{b_n} \right| \xrightarrow{n\to\infty} \infty \Rightarrow$ Diverges Σa_n

$\left| \frac{a_n}{b_n} \right| \xrightarrow{n\to\infty} c \neq 0 \Rightarrow$

May 8-12:41 PM

$$\frac{d}{dx}\left[\ln(2x+1)\right] = \frac{2}{2x+1}$$

$$\int \frac{x^3\,dx}{\sqrt{x^2-1}}$$

$x = \sec\theta$

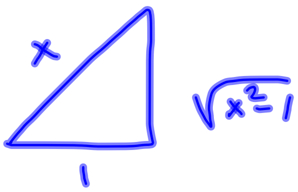
$dx = \sec\theta\tan\theta\,d\theta$

$$= \int \frac{\sec^3\theta\,\sec\theta\tan\theta}{|\tan\theta|}\,d\theta$$

Assume $\tan\theta > 0$. Otherwise, just take the negative.

$$= \int \sec^4\theta\,d\theta = \int \sec^2\theta(\tan^2\theta+1)\,d\theta$$

$$= \int \tan^2\theta\sec^2\theta\,d\theta + \int \sec^2\theta\,d\theta$$



$$= \frac{1}{3}\tan^3\theta + \tan\theta + C$$

$$= \frac{1}{3}(x^2-1)^{\frac{3}{2}} + (x^2-1)^{\frac{1}{2}} + C$$

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$$\int \frac{x^3\,dx}{\sqrt{x^2-1}} =$$

$u = x^2 - 1$

$x^2 = u + 1$

$du = 2x\,dx$

$$= \frac{1}{2}\int \frac{x^2 \cdot 2x\,dx}{\sqrt{x^2-1}} = \frac{1}{2}\int \frac{(u+1)\,du}{\sqrt{u}}$$

$$\sum \frac{1}{n^2+n} = \sum \left[\frac{1}{n} - \frac{1}{n+1}\right]$$

May 8-12:53 PM

$$\binom{-\frac{1}{3}}{0} = 1$$

$$\binom{-\frac{1}{3}}{1} = \frac{\left(-\frac{1}{3}-0\right)}{1!} = -\frac{1}{3}$$

$$\binom{-\frac{1}{3}}{2} = \frac{\left(-\frac{1}{3}-0\right)\left(-\frac{1}{3}-1\right)}{2!} = \frac{\left(-1/3\right)\left(-\frac{4}{3}\right)}{2} = \frac{2}{9}$$

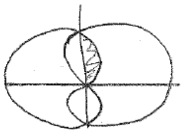
$$\binom{-\frac{1}{3}}{3} = \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)}{3!} = \frac{\left(-\frac{1}{3}-0\right)\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3!} = \frac{-\frac{28}{27}}{6}$$

$$= \frac{-\frac{14}{27}}{3} = -\frac{14}{81}$$

15 11.6

S11.5 12 $r=2(1+\cos\theta)$
$r=2(1-\cos\theta)$

All kinds of symmetry



$$4\int_0^{\frac{\pi}{2}} \frac{(2(1-\cos\theta))^2}{2}\,d\theta$$

$$\int_{\frac{\pi}{2}}^{\frac{3\cdot\pi}{2}} \frac{1}{2}\cdot(2\cdot(1+\cos(x)))^2\,dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}\cdot(2\cdot(1-\cos(x)))^2\,dx$$

$$6\pi - 16$$

$$4\cdot\int_0^{\frac{Pi}{2}} \frac{1}{2}\cdot(2\cdot(1-\cos(x)))^2\,dx$$

$$6\pi - 16$$