$$\lim_{x\to 0} \frac{e^x - e^{-x}}{x} \overset{L'H}{=} \lim_{x\to 0} (e^x + e^{-x}) = e^0 + \frac{1}{e^0} = 2$$

$$\frac{e^x - e^{-x}}{x} = \frac{1}{x}\left[\sum_0^\infty \frac{x^k}{k!} - \sum_0^\infty (-1)^k \frac{x^k}{k!}\right]$$

$$= \frac{1}{x}\sum_{k=0}^\infty \frac{2x^{2k+1}}{(2k+1)!} = \sum_{k=0}^\infty \frac{2x^{2k}}{(2k+1)!} \xrightarrow{x\to 0} 2$$

$$\frac{1}{x}\left[x + \frac{2x^3}{3!} + \frac{2x^5}{5!} + \dots\right] = 1 + \frac{2x^2}{3!} + \frac{2x^4}{5!} + \dots \xrightarrow{x\to 0} 2$$

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10.10 cont'd

$$\binom{n}{0} = 1 \quad \binom{n}{2} = \frac{n(n-1)}{2!}$$

$$(x^2+1)^{-\frac{1}{3}}$$

$$\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{k!}$$

$$= \sum_{k=0}^{\infty} \binom{-1/3}{k} (x^2)^k$$

$$(x+1)^n = \sum_{k=0}^{\infty} \binom{n}{k} x^k$$

$$= 1 + \sum_{k=1}^{\infty} \binom{n}{k} x^k$$

$$\binom{-\frac{1}{3}}{0} = 1$$

$$\binom{-\frac{1}{3}}{1} = -\frac{1}{3}$$

$$\binom{-\frac{1}{3}}{2} = \frac{\left(-\frac{1}{3}\right)\left(-\frac{1}{3}-1\right)}{2!} = \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2}$$

$$\binom{-\frac{1}{3}}{3} = \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)}{3!}$$

$$(x^2+1)^{-\frac{1}{3}} = 1 - \frac{1}{3}x^2 + \frac{2}{9}(x^2)^2 - \frac{14}{81}(x^2)^3 + \cdots$$

$$= 1 - \frac{1}{3}x^2 + \frac{2}{9}x^4 - \frac{14}{81}x^6 + \cdots$$

10.x

$$\lim_{n\to\infty} \left(1 - \frac{3}{n}\right)^n = ?$$

$$-\frac{3}{n} = -3\left(\frac{1}{n}\right)$$

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$$y=\left(1-\frac{3}{n}\right)^n \implies \ln y = \ln\left(\left(1-\frac{3}{n}\right)^n\right)$$

$$= n\ln\left(1-\frac{3}{n}\right) = \frac{\ln\left(1-\frac{3}{n}\right)}{\frac{1}{n}} \xrightarrow[\text{L'H}]{n\to\infty} \frac{\frac{\frac{3}{n^2}}{1-\frac{3}{n}}}{-\frac{1}{n^2}}$$

$$= \frac{\frac{3}{n^2}\cdot\left(-\frac{n^2}{1}\right)}{1-\frac{3}{n}} \xrightarrow{n\to\infty} -3 =$$

$$\lim_{n\to\infty} \ln(y) = -3$$

$$\lim_{n\to\infty} y = e^{-3}$$

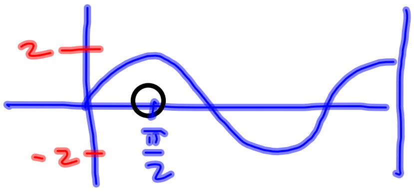
$$\boxed{\left(1+\frac{1}{x}\right)^x \xrightarrow{x\to\infty} e}$$

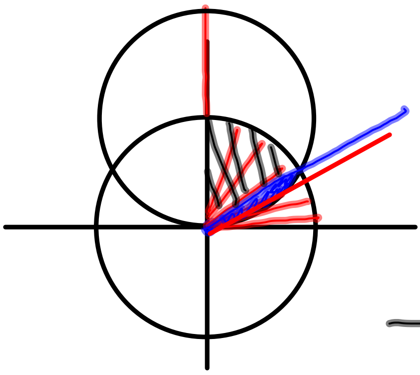
$$\left(1-\frac{3}{n}\right)^n = \left(1+\left(-\frac{3}{n}\right)\right)^n = \left(1+\left(\frac{1}{-\frac{n}{3}}\right)\right)^{\left(-\frac{n}{3}\right)(-3)}$$

$$= \left(\boxed{\left(1+\frac{1}{-\frac{n}{3}}\right)^{-n/3}}\right)^{-3} \xrightarrow{n\to\infty} e^{-3}$$

11.5 #10

Area shared by $r=1$, $r=2\sin\theta$

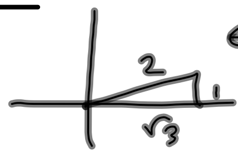




$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\theta = \arcsin\left(\frac{1}{2}\right)$



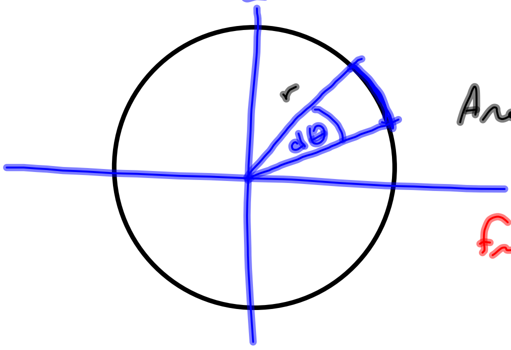
$= \frac{\pi}{6}$

$$\cancel{2\left[\int_0^{\frac{\pi}{6}} 2\sin\theta\, d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 1\, d\theta\right]}$$

Not the area in polar coords

$$\int \frac{1}{2} r^2\, d\theta$$

$$2\left[\frac{1}{2}\int_0^{\frac{\pi}{6}} 4\sin^2\theta\, d\theta + \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 1^2\, d\theta\right]$$



Area of rep. slice is $\frac{1}{2}r^2 d\theta$

from area of sector of circle.

$$A = \pi r^2 = \frac{1}{2}\underbrace{(2\pi)}_{\theta} r^2$$

$r = 1 \qquad \sqrt{x^2+y^2} = 1$

$$x^2+y^2 = 1$$

$$y = \pm\sqrt{1-x^2}$$

$r = 2\sin\theta$

$\sqrt{x^2+y^2} = 2\frac{y}{r} \qquad = 2\frac{y}{\sqrt{x^2+y^2}}$

$x^2+y^2 = 2y$

$x^2+y^2-2y+1^2 = 1^2$

$x^2+(y-1)^2 = 1$

$(y-1)^2 = 1-x^2$

$y-1 = \pm\sqrt{1-x^2}$

$y = 1\pm\sqrt{1-x^2}$

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$$2\left[\int_0^{\sqrt{3}} \sqrt{1-x^2}\,dx - \int_0^{\sqrt{3}} \left(1+\sqrt{1-x^2}\right)dx\right]$$

$$= 2\left[\frac{1}{2}\int_0^{\frac{\pi}{6}} 4\sin^2\theta\,d\theta + \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 1\,d\theta\right]$$

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