§10.6 $\sum_{n=2}^{\infty} (-1)^n \frac{4}{\ln(n)}$

How many terms 'til error is < .01?

$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n!}{2^n}$

(a) Just crank out terms, 'til one is smaller than .01

(1) Solve $|a_n| < .01$ algebraically.

(1) If you can solve analytically, do so.

(2) If you can't then crank.

$$\frac{4}{\ln(n)} \overset{\text{want}}{<} .01$$

$$\frac{4}{.01} < \ln(n)$$

$$400 < \ln(n)$$

$$e^{400} < n$$

Method (a) would've sucked.

55 $\sum (-1)^{n+1} \frac{1}{(n+3\sqrt{n})^3}$ can be done either way

Error < .001

$$|a_n| = \frac{1}{(n+3\sqrt{n})^3} \overset{\text{want}}{<} .001$$

$$1000 < (n+3\sqrt{n})^3$$

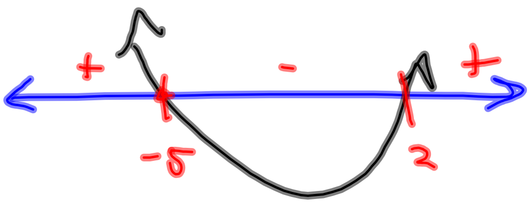
$$(n+3\sqrt{n})^3 > 1000$$

$$n+3\sqrt{n} > \sqrt[3]{1000} = 10$$

$$n+3\sqrt{n} - 10 > 0$$

$$u^2+3u-10 > 0$$

$$(u+5)(u-2) > 0$$



So $n > 4$ does it.

Not sure if I

$u > 2$

$u = \sqrt{n}$

$u^2 = 2^2 = n$

$\boxed{n = 4}$

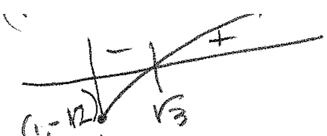
$\Rightarrow n + 3\sqrt{n} - 10 > 0$

$u^2 + 3u - 10 > 0$

$(u+5)(u-3) > 0$

$u = -5$ OR $u = 3$ $\Rightarrow$ $n = \sqrt{3} \approx 1.$...

use $\boxed{n = 2.}$



$\rightarrow u = 3 = \sqrt{n} \Rightarrow 9 = n$, dummy!

This was crap. Factored wrong and then apparently used $u = n^2$ instead of $u = \sqrt{n}$ when I back substituted. Man

(55) $\sum (-1)^{n+1} \frac{1}{(n+3\sqrt{n})^3}$

Method (2)

$|a_1| = \frac{1}{(1+3)^3} = \frac{1}{4^3} = \frac{1}{64}$

$|a_2| = \frac{1}{(2+3\sqrt{2})^3} \approx .0041$

$|a_3| \approx .0018$

$|a_4| \approx .001$ Good.

```
      .0041105031
Y1(3
      .0018162261
Y1(4
             .001
1/216
      .0046296296
```

So this is good, when we're right on.

If it's monotone, STRICTLY decreasing, then this works ok

$$\left| \sum_{n=4}^{\infty} (-1)^n \frac{1}{(n+3\sqrt{n})^3} \right| < |a_4|$$

§10.7 Power Series.

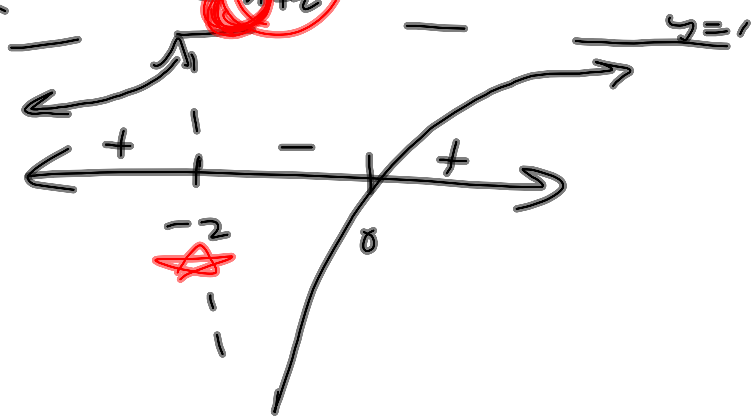
$\sum_{n=1}^{\infty} \frac{n x^n}{n+2}$: $\left|\frac{a_{n+1}}{a_n}\right|$

$= \left| \frac{(n+1)x^{n+1}}{n+3} \cdot \frac{n+2}{n x^n} \right| = \left| \frac{n^2(1+\frac{1}{n})(1+\frac{2}{n})}{n^2(1+\frac{3}{n})} \right| |x|$

$\xrightarrow{n \to \infty} |x| \overset{\text{want}}{<} 1$ $\boxed{R = 1}$

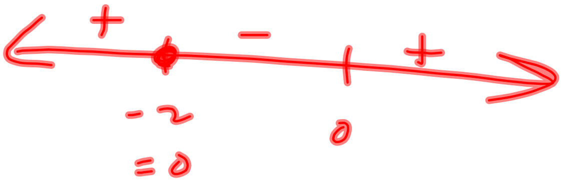
$x = 1$: $\sum_{n=1}^{\infty} \frac{n}{n+2} \not\to$

$x = -1$: $\sum \frac{n}{n+2}(-1)^n \not\to$

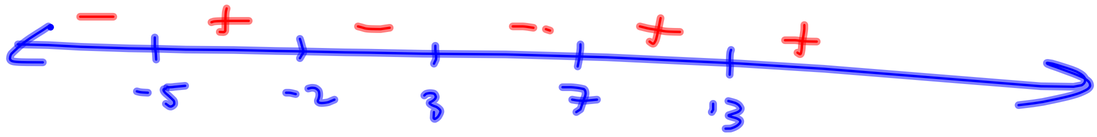


$\boxed{\frac{x}{x+2}}$

$\boxed{x(x+2)}$



Domain of $\sqrt{\dfrac{(x+2)^3(x-13)^2}{(x-3)^2(x+5)^{11}(x-7)}}$



want ≥ 0.

$(-5,-2] \cup (7,13] \cup [13,\infty)$

$-5, 7 \notin D$ $\left(\dfrac{\sim}{0}\right)$

$$\sum_{1}^{\infty} (-1)^n \frac{(x+2)^n}{n}$$

$$\left| \frac{(x+2)^{n+1}}{n+1} \cdot \frac{n}{(x+2)^n} \right| = \frac{n}{n+1} |x+2|$$

$|x-(-2)|$

$\xrightarrow{n \to \infty}$ $\underline{|x+2| < 1}$

$-1 < x+2 < 1$ $\quad R = 1$

$-3 < x < -1$ $\quad I = [-1, 3)$

$x = -3$: $\quad \sum \frac{1}{n} \not\rightarrow$

$x = -1$: $\quad \sum \frac{(-1)^n}{n} \longrightarrow$

$$\sum (-1)^n \frac{(e^x - 2)^n}{n}$$

$$\boxed{\sum (e^x - 5)^n}$$

$$\left| \frac{(e^x-5)^{n+1}}{(e^x-5)^n} \right| = |e^x - 5| < 1$$

$$\sum \frac{(e^x-5)^n}{n}$$

$$\sum \frac{(-1)^n (e^x-5)^n}{n}$$

$-1 < e^x - 5 < 1$

$4 < e^x < 6$

$\ln 4 < x < \ln 6$

check endpts. No go

$I = (\ln 4, \ln 6)$

10.8

$\frac{x^2}{x+7}$ is about as hard as it can get.

$$\frac{x^2}{x+1}$$

$$\frac{x+3}{x+7} = (x+3)\left(\frac{1}{7\left(\frac{x}{7}+1\right)}\right)$$

$$= \frac{(x+3)}{7}\left(\frac{1}{1-\left(-\frac{x}{7}\right)}\right)$$

$$= \left(\frac{x+3}{7}\right)\left(1 - \frac{x}{7} + \left(\frac{x}{7}\right)^2 + \cdots\right)$$

$$\frac{x^2}{1-(-x)} = x^2 \sum_{n=1}^{\infty} (-x)^{n-1}$$

$$\boxed{\frac{1}{1-x} = \sum_{n=1}^{\infty} x^{n-1}} = \sum_{n=0}^{\infty} x^n$$

10.9 Slip-in:

Series for $x \ln(2x+1)$

$$\frac{d}{dx}\ln(2x+1) = \frac{2}{2x+1} = \frac{2}{1-(-2x)}$$

$$= 2\left[1 - 2x + (2x)^2 - (2x)^3 + \cdots\right]$$

$$\Rightarrow \ln(2x+1) = 2\left[\underline{x - x^2 + \frac{4}{3}x^3 - 2x^4 + \cdots}\right] + C$$

by integrating, term by term

$$\underline{\ln(2(0)+1) = \ln 1 = 0 = C}$$

So $x\ln(2x+1) =$

$$2\left[x^2 - x^3 + \frac{4}{3}x^4 - 2x^5 + \cdots\right]$$

$$= 2x^2 - 2x^3 + \frac{8}{3}x^4 - 4x^5 + \cdots$$