§10.3 How many terms to keep

$$S = \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^4} \text{ w/in } \frac{1}{100} \text{ of actual?}$$

want $|S - S_n| < \frac{1}{100}$.

$$|R_n| < \frac{1}{100}$$

$$\sum_{k=n+1}^{\infty} a_k = \sum_{k=n+1}^{\infty} \frac{1}{n(\ln n)^4}$$

Integral Test does NOT work on conditionally convergent series.

$$\sum_{1}^{\infty} (-1)^n \frac{1}{n}$$

$$\int_1^{\infty} \frac{dx}{x} \quad \cancel{\exists}$$

Integral test needs

a_n's the same sign.

$|a_n| \xrightarrow{n \to \infty} 0$

$\frac{d}{dn}(a_n) < 0$ or make a case for decreasing.

$$\sum_{k=n+1}^{\infty} a_k = \sum_{k=n+1}^{\infty} \frac{1}{n(\ln n)^4} \leq \int_n^{\infty} \frac{1}{x(\ln x)^4}\, dx \quad \text{want } < .01$$

Find n.

$$\Longrightarrow \int_n^{\infty} (\ln x)^{-4} \left(\frac{1}{x}\, dx \right)$$

$$= \lim_{b\to\infty} \left[\frac{(\ln x)^{-3}}{-3} \right]_n^b$$

$$= -\frac{1}{3}\left[\lim_{b\to\infty} \frac{1}{(\ln b)^3} - \frac{1}{(\ln n)^3} \right]$$

$$= \frac{1}{3(\ln n)^3} \overset{\text{want}}{<} .01 = \frac{1}{100}$$

$$\longrightarrow \left(\frac{1}{3}\right)\left(\frac{100}{1}\right) < (\ln n)^3$$

$$(\ln n)^3 > \frac{100}{3}$$

$$\ln n > \sqrt[3]{\frac{100}{3}}$$

$$n > e^{\sqrt[3]{\frac{100}{3}}} \approx 24.98555727$$

So, $n = 25$ does it.

```
e^((100/3)^(1/3)
)
        24.98555727
```

§10.4 Comparison tests.

Direct Comparison

(1) Show $\sum_{n=1}^{\infty} \frac{n^2-n-1}{n^4+n^3+11}$ converges.

$$a_n = \frac{n^2-n-1}{n^4+n^3+11} \le \frac{n^2}{n^4} = \frac{1}{n^2} = b_n$$

& $\sum b_n$ passes $p=2$-test: converges.

(2) Show $\sum_{n=1}^{\infty} \frac{n^2+n+1}{n^3-n^2-11}$ diverges.

$$a_n = \frac{n^2+n+1}{n^3-n^2-11} \ge \frac{n^2}{n^3} = \frac{1}{n} = b_n$$ & $\sum b_n$ diverges (Harmonic)

$p=1$-test fail.

Bonus! Show that

$\sum \frac{n^2+n+1}{n^4-n^2-11}$ converges BY DIRECT comparison.

$$a_n = \frac{n^2+n+1}{n^4-n^2-11} \le \frac{n^2+\frac{1}{2}n^2+\frac{1}{4}n^2}{n^4-\frac{1}{2}n^4-\frac{1}{4}n^4} = \frac{\frac{7}{4}n^2}{\frac{1}{4}n^4} = \frac{7}{n^2} = b_n$$

etc.

Limit Comparison. See above examples.

Bonus for Direct is just "regular" limit comparison.

§10.5 Ratio & Root Test

$$\sum \left(\frac{2n+3}{5n+4}\right)^n$$

Root $\sqrt[n]{|a_n|} = \frac{2n+3}{5n+4} \xrightarrow{n\to\infty} \frac{2}{5}$

Ratio $\left|\frac{a_{n+1}}{a_n}\right| = \left(\frac{2(n+1)+3}{5(n+1)+4}\right)^{n+1}\left(\frac{5n+4}{2n+3}\right)^n$

$$= \left(\frac{(2n+5)^{n+1}}{(5n+9)^{n+1}}\right)\left(\frac{(5n+4)^n}{(2n+3)^n}\right)$$

$$= \frac{\left(n\left(2+\frac{5}{n}\right)\right)^{n+1}}{\left(n\left(5+\frac{4}{n}\right)\right)^{n+1}} \cdot \frac{n^n\left(5+\frac{4}{n}\right)^n}{n^n\left(2+\frac{3}{n}\right)^n}$$

$$= \frac{n^{n+1}}{n^{n+1}} \cdot \frac{n^n}{n^n} \cdot \frac{\boxed{\left(2+\frac{5}{n}\right)^{n+1}}}{\left(5+\frac{4}{n}\right)^{n+1}} \cdot \frac{\left(5+\frac{4}{n}\right)^n}{\left(2+\frac{3}{n}\right)^n}$$

Teacher sucks

$$2^{n+1}\left(1+\frac{5}{2n}\right)^{n+1} = 2^{n+1}\boxed{\left(1+\frac{1}{\boxed{\left(\frac{2n}{5}\right)}}\right)^{\frac{2n}{5}\cdot\frac{5}{2}+1}}$$

$\left(1+\frac{1}{x}\right)^x \xrightarrow{x\to\infty} e$

$(1+x)^{\frac{1}{x}} \xrightarrow{x\to 0} e$

$n+1 = \frac{2n}{5}\cdot\frac{5}{2}+1$

$\sum \frac{n!}{10^n}$ Diverges $a_n \not\xrightarrow{n\to\infty} 0$

$$\boxed{\frac{10!}{10^{10}}}$$

$\sum$

$$\frac{11!}{10^{11}} = \frac{11}{10} \cdot \boxed{\frac{10!}{10^{10}}}$$

$$\frac{12!}{10^{12}} = \frac{12}{10} \cdot \frac{11}{10} \cdot \boxed{\frac{10!}{10^{10}}}$$

ratio test nails this type.

Root test:

$\sqrt[n]{\frac{n!}{10^n}}$ Nowheah, babee.

Ratio Test $(n+1)! = (n+1) \cdot n!$

$$\left| \frac{a_{n+1}}{a_n} \right| = \frac{(n+1)!}{10^{n+1}} \cdot \frac{10^n}{n!}$$

$$= \frac{n+1}{10} \xrightarrow{n\to\infty} \infty$$

$\sum \frac{10^n}{n!}$ converges by ratio test.

$\sum_{n=1}^{\infty} \left(1-\frac{1}{3n}\right)^n \qquad a_n \not\to 0$

$a^{bc} = (a^b)^c$

$\boxed{a_n = \left(1-\frac{1}{3n}\right)^n} = \left(1-\frac{1}{3n}\right)^{\frac{3n}{3}}$

$= \left(\left(1-\frac{1}{3n}\right)^{3n}\right)^{\frac{1}{3}}$

$=$

was shooting for an "e" deal.e and then Adam & I saw the $1-\frac{1}{3n}$ when $1+\frac{1}{3n}$ is needed.

$y = \left(1-\frac{1}{3n}\right)^n$

$\to \ln y = \ln\left(\left(1-\frac{1}{3n}\right)^n\right)$

$= n\ln\left(1-\frac{1}{3n}\right) \xrightarrow{n\to\infty} \infty\cdot 0$

$= \dfrac{\ln\left(1-\frac{1}{3n}\right)}{\frac{1}{n}} \xrightarrow[L'H]{n\to\infty} \dfrac{\frac{-\frac{1}{3n^2}}{1-\frac{1}{3n}}}{-\frac{1}{n^2}}$

$= \dfrac{\frac{-\frac{1}{3n^2}}{\frac{3n-1}{3n}}}{-\frac{1}{n^2}} = \dfrac{-\frac{1}{3n^2}\left(\frac{3n}{3n-1}\right)}{-\frac{1}{n^2}}$

$= -\frac{1}{3n^2}\left(-\frac{n^2}{1}\right)\left(\frac{3n}{3n-1}\right) \xrightarrow{n\to\infty} \frac{1}{3}$

$\ln y = \frac{1}{3}$

$y = e^{\frac{1}{3}}$

Apr 29-12:51 PM