10.1

$a_n = \frac{n^2 2n+1}{n-1} \not\to$

$n^{\frac{1}{n}} \xrightarrow{n\to\infty} 1$

$a_n = \left(\frac{n+1}{2n}\right)\left(1-\frac{1}{n}\right)$

$\left(\frac{3}{n}\right)^{\frac{1}{n}} = \frac{3^{\frac{1}{n}}}{n^{1/n}}$

$3^x = e^{\ln(3^x)}$
$= e^{x\ln 3}$
$= e^{(\ln 3)x}$

Prove that $3^{\frac{1}{n}}$ converges.

3^x is increasing
$\frac{1}{n}$ is decreasing

$3^{\frac{1}{n}}$ decreasing

$\frac{d}{dx}\left[3^{\frac{1}{x}}\right]$
$= (\ln 3)\, 3^{\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right) < 0$

$\frac{d}{dx}[e^u] = u'(x)e^u$

$\frac{1}{n} > 0$

$\frac{d}{dx}\left[e^{(\ln 3)x}\right] = (\ln 3)\boxed{e^{(\ln 3)x}}$

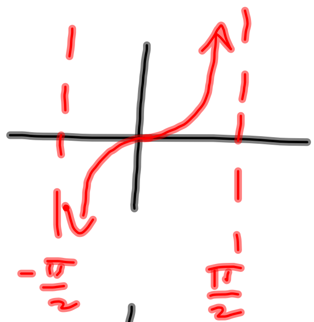
$3^{\frac{1}{n}} > 3^0 = 1$ i.e., $3^{\frac{1}{n}}$ bdd below by 1.

Decreasing sequence bdd below converges.

$3^x = 2^{\log_2(3^x)}$
$= 2^{(\log_2(3))x}$

$e^{x\ln 3}$
$= e^{\ln(3^x)}$
$= 3^x$

$\{\arctan(n)\}$



$$\int \frac{dx}{x^2+1}$$

$\frac{1}{x^2+1}$ $\frac{d}{dx}(\arctan x)$

$\sum \frac{1}{n^2} \rightarrow$ $\sum \frac{1}{n} \not\rightarrow$

$\sum\left(\frac{1}{n^2}+\frac{1}{n}\right) \not\rightarrow$

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§10.2

$$\sum_{n=0}^{\infty} \frac{5}{4^n} \qquad 5\left(\frac{1}{1-\frac{1}{4}}\right) = 5\left(\frac{1}{3/4}\right) = \frac{20}{3}$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \forall\ |x|<1$$

$$\sum_{n=0}^{\infty} \left(\frac{5}{4}\right)^n \qquad \sum (-1)^n \frac{5}{4^n}$$

$$1.\overline{37} = 1 + \sum_{n=1}^{\infty} (37)\left(\frac{1}{100}\right)^n = 1 + 37\left(\frac{1}{\frac{99}{100}}\right)$$

$$= 1 + \frac{3700}{99}$$

off by factor of 100

$.37 + .0037 + .000037 + \cdots$

$$\frac{37}{100} + \frac{37}{100^2} + \frac{37}{100^3} + \cdots$$

$$1 + \sum_{n=1}^{\infty} \boxed{(37)\left(\frac{1}{100}\right)}\left(\frac{1}{100}\right)^{n-1}$$

$\boxed{\text{Fixed}}$

$a = \frac{37}{100}$ $\qquad r = \frac{1}{100}$

$$\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$$ telescopes $\sum_1 \frac{1}{n^2+n}$

$$\sum \frac{1}{n} - \sum \frac{1}{n+1} \qquad \sum\left(\frac{1}{n}-\frac{1}{n^2}\right)$$

S $\quad 1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\cdots+\left(\frac{1}{n}-\frac{1}{n+1}\right)+\cdots$

S_n $\qquad S_n = 1-\frac{1}{n+1}$

$S_3 = 1-\frac{1}{4}$ $\qquad S_n = \sum_{k=1}^{n} a_k$

$S_4 = 1-\frac{1}{5}$

§10.3 Integral Test

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

$$\int_2^{\infty} \frac{dx}{x \ln x}$$

$$\lim_{b\to\infty} \ln(\ln x)\Big]_2^b \notin \mathbb{R}$$

$$\lim_{b\to\infty} \left(\ln(\underbrace{\ln b}_{\to\infty})\right) = \infty$$

$$\int_2^{\infty} (\ln x)^{-4} \cdot \frac{1}{x}\,dx \qquad \int_2^{\infty} \frac{dx}{x(\ln(x))^4}$$

$$\lim_{b\to\infty} \left[-\frac{1}{3}(\ln x)^{-3}\right]_2^b = \underbrace{\lim_{b\to\infty} -\frac{1}{3}\left(\frac{1}{\ln(b)^3}\right)}_{\to 0} - \left(-\frac{1}{3}(\ln 2)^{-3}\right)$$

$$\int_2^{\infty} \frac{dx}{x(\ln x)^2}$$

$\sum \frac{1}{x(\ln x)^2}$ converges SLOWLY.