$$(x+c)^2+y^2 = 4a^2 - 4a\sqrt{(x-c)^2+y^2} + (x-c)^2+y^2$$

$$x^2+2xc+c^2+y^2 = 4a^2-4a\sqrt{\quad} + x^2-2cx+c^2+y^2$$

$$\cancel{x^2}+2xc+\cancel{c^2}+\cancel{y^2} = 4a^2-4a\sqrt{\quad} + \cancel{x^2}-2cx+\cancel{c^2}+\cancel{y^2}$$

$$4cx = 4a^2-4a\sqrt{\quad}$$

$$cx = a^2 - a\sqrt{(x-c)^2+y^2}$$

$$cx-a^2 = -a\sqrt{\quad}$$

$$a^2-cx = a\sqrt{\quad}$$

$$a^4-2a^2cx+c^2x^2 = a^2(x^2-2cx+c^2+y^2)$$

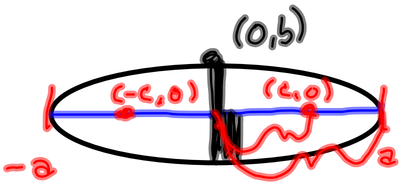
$$a^4\underline{-2a^2cx}+c^2x^2 = a^2x^2\underline{-2a^2cx}+a^2c^2+a^2y^2$$

$$c^2x^2-a^2x^2-a^2y^2 = a^2c^2-a^4 = a^2(c^2-a^2)$$

$$x^2(c^2-a^2)-a^2y^2 = a^2(c^2-a^2)$$

$$\frac{x^2}{a^2}-\frac{y^2}{c^2-a^2}=1$$

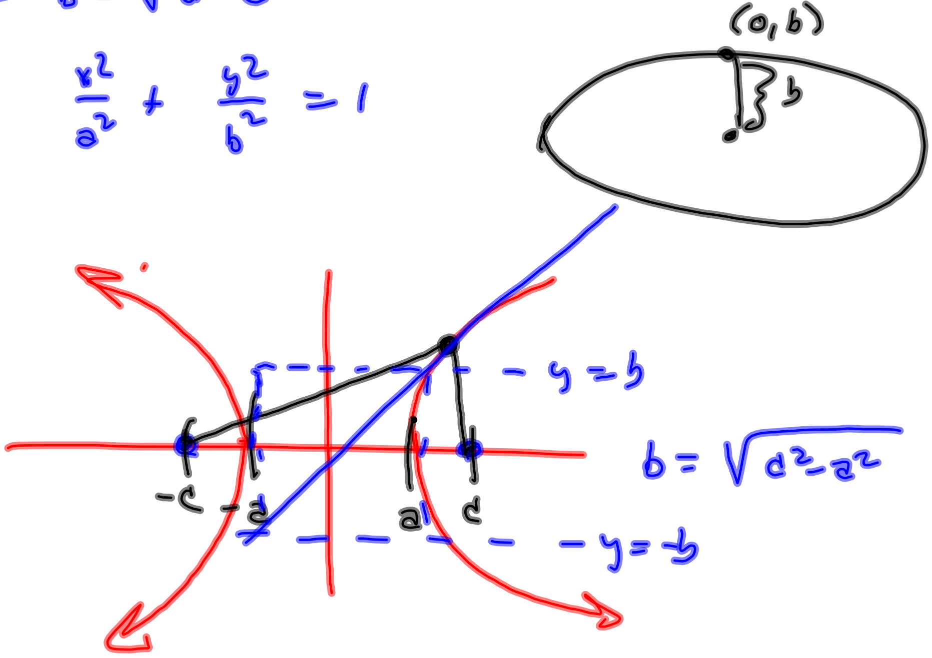
$$c<a \Rightarrow c^2-a^2<0$$



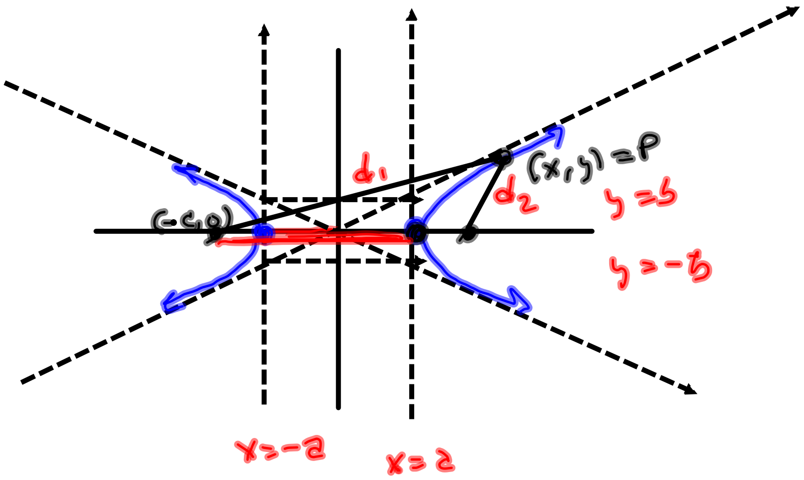
$$\frac{x^2}{a^2} + \frac{y^2}{a^2-c^2} = 1$$

Let $b = \sqrt{a^2-c^2}$ Then

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$b = \sqrt{c^2-a^2}$



2

$$d_1 - d_2 = \text{constant} = 2a$$

$$\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} = 2a$$

$$\sqrt{(x+c)^2 + y^2} = \sqrt{(x-c)^2 + y^2} + 2a$$

$$x^2 + 2cx + c^2 + y^2 = x^2 - 2cx + c^2 + y^2 + 4a\sqrt{} + 4a^2$$

$$\cancel{x^2} + 2cx + \cancel{c^2} + \cancel{y^2} = \cancel{x^2} - 2cx + \cancel{c^2} + \cancel{y^2} + 4a\sqrt{} + 4a^2$$

$$4cx - 4a^2 = 4a\sqrt{(x-c)^2 + y^2}$$

$$cx - a^2 = a\sqrt{(x-c)^2 + y^2}$$

$$c^2x^2 - 2a^2cx + a^4 = a^2(x^2 - 2cx + c^2 + y^2)$$

$$c^2x^2 - 2a^2cx + a^4 = a^2x^2 - 2a^2cx + a^2c^2 + a^2y^2$$

$$c^2x^2 \cancel{- 2a^2cx} + a^4 = a^2x^2 \cancel{- 2a^2cx} + a^2c^2 + a^2y^2$$

$$c^2x^2 - a^2x^2 - a^2y^2 = a^2c^2 - a^4 = a^2(c^2 - a^2)$$

$$\frac{x^2(c^2-a^2)}{a^2(c^2-a^2)} - \frac{a^2y^2}{a^2(c^2-a^2)} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{c^2-a^2} = 1$$ is correct. $c > a$.

$$\frac{x^2}{a^2} + \frac{y^2}{a^2-c^2} = 1$$ is equivalent, so you THINK it's an ellipse, right? Wrong.

$c > a$ for the hyperbola

Let $b = \sqrt{c^2-a^2} \Rightarrow$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

§11.6 #s 11, 13, 19, 23, 25, 33, 37, 49, 62, 64

#s 9–16 Find focus & directrix
sketch.

$x^2 = 6y$

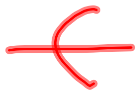
$y = \frac{x^2}{6}$

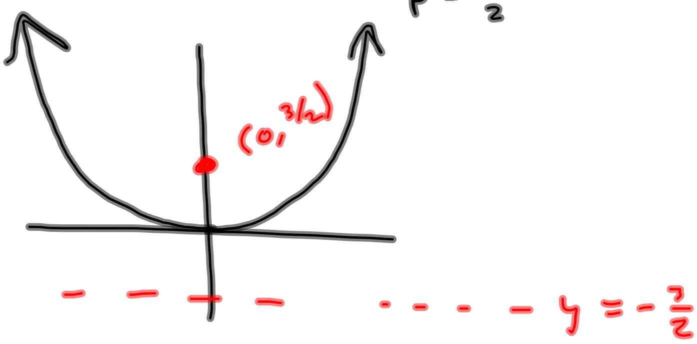
$4p = 6$

$p = \frac{3}{2}$

$y = \frac{x^2}{4p}$

$x = \frac{y^2}{4p}$





#17 – 24 Ellipses

$$9x^2 + 10y^2 = 90$$

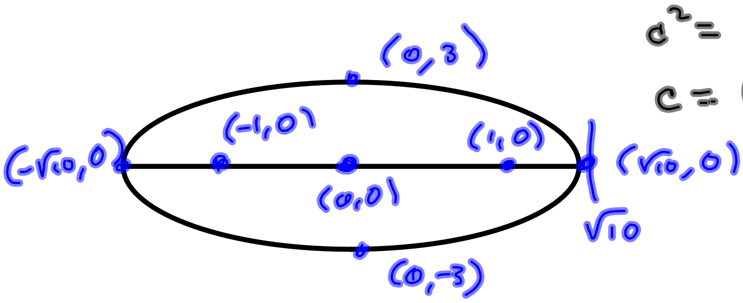
$$\frac{x^2}{10} + \frac{y^2}{9} = 1$$

$a = \sqrt{10}$

$b = 3$

$c^2 = a^2 - b^2$

$c = \sqrt{a^2 - b^2} = \sqrt{10 - 9} = 1$



$$9x^2 + 6y^2 + 36y = 0$$

$$9x^2 + 6(y^2 + 6y \quad) = 0$$

Factor out a 3, dummy.

$$3x^2 + 2(y^2 + 6y + 3^2) = 0 + 18$$

$$3x^2 + 2(y+3)^2 = 18$$

$$(h,k) = (0,-3)$$

$$\frac{x^2}{6} + \frac{(y+3)^2}{9} = 1$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$a^2 = 6$

$b^2 = 9$

$$b^2 - a^2 = 9 - 6 = 3 = c^2$$

$$c = \sqrt{3}$$