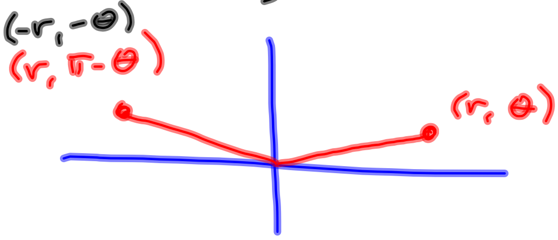
§11.4 #s 1, 6, 9, 10, 13, 14, 17, 18, 25, 26

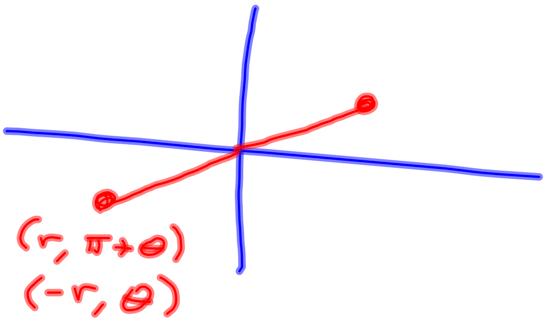
§11.5 #s 2, 5, 8, 10-12, 16, 23, 29, 30

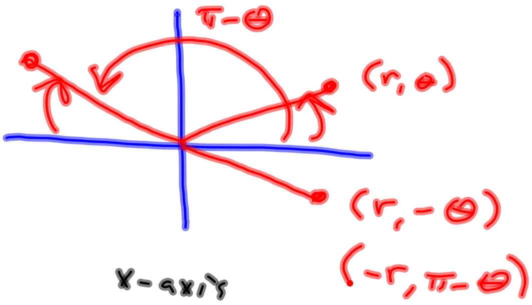
§11.3 2, 6, 8, 14, 18, 26, 36, 40, 46, 66

Symmetry of a polar graph



y-axis.





x-axis

① $r = 1 + \cos\theta$

$(r, -\theta)$: $1 + \cos(-\theta) = 1 + \cos\theta = r$ equivalent.

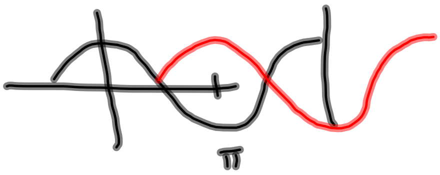
(i) x-axis ✓

$r = 1 + \cos(\pi - \theta)$

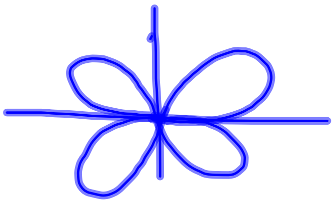
$= 1 + \cos(-(\theta - \pi))$

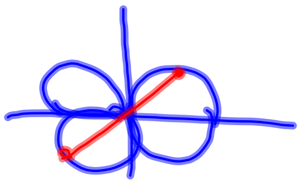
$= 1 + \cos(\theta - \pi) = 1 - \cos\theta$ Not Equiv. No.

(ii) y-axis No



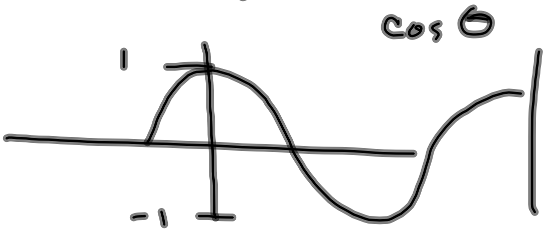
⟹ (iii) origin No.

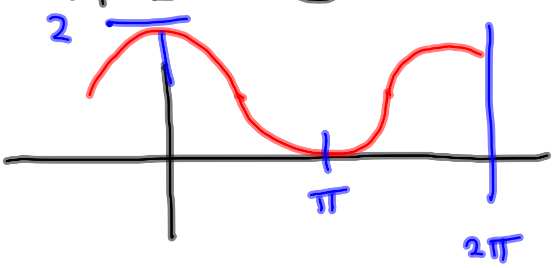




one & not the other ⟹ No symmetry thru origin.

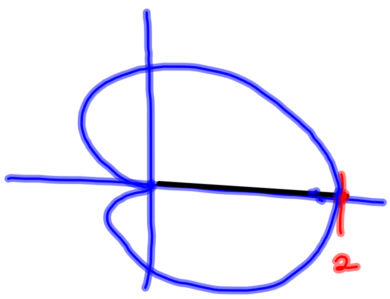
Let's graph it





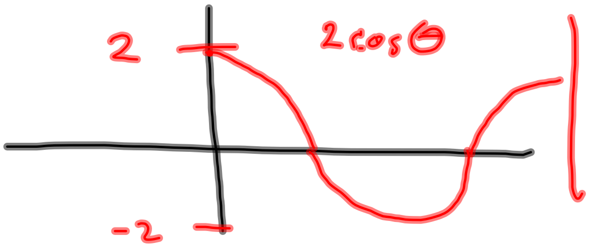
Use as "table" for polar graph

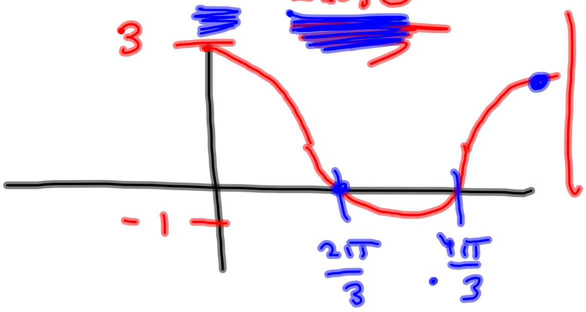
$r = 1 + \cos\theta$



Symmetry about x-axis confirmed (mod my skill)

$r = 1 + 2\cos\theta$

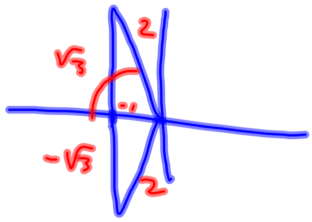




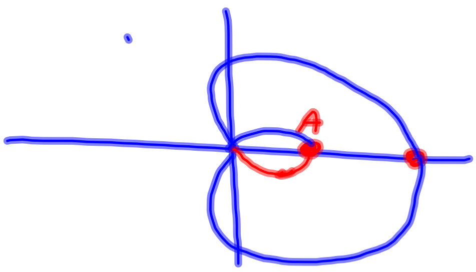
$1 + 2\cos\theta = 0$

$2\cos\theta = -1$

$\cos\theta = -\frac{1}{2}$



$\pi \pm \frac{\pi}{3} \nearrow \frac{4\pi}{3}$, $\searrow \frac{2\pi}{3}$

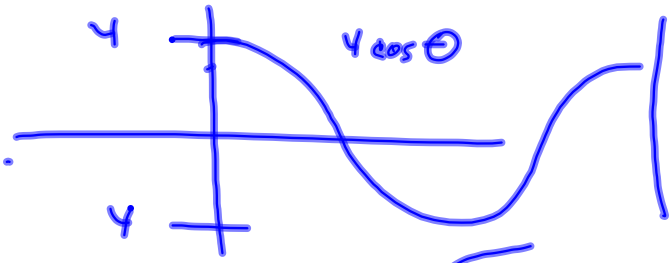


$A = (-1, \pi)$

$r^2 = 4\cos\theta$ → $r = 2\sqrt{\cos\theta}$

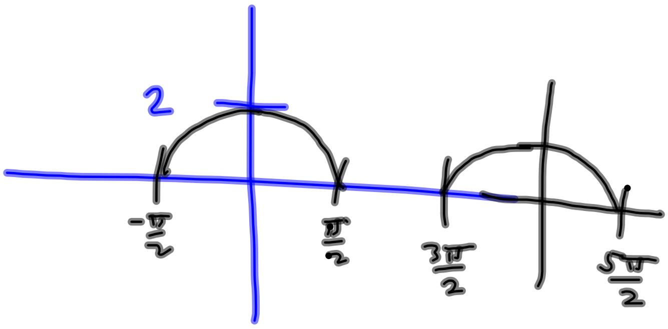
$r^2 = 4\cos\theta$ → $r = -2\sqrt{\cos\theta}$

$4\cos\theta$

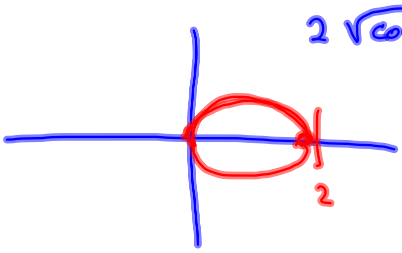


$2\sqrt{\cos\theta}$

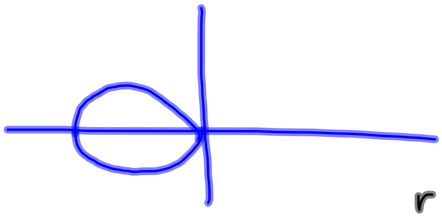
Keep $\cos\theta \geq 0$ for $\sqrt{\cos\theta}$



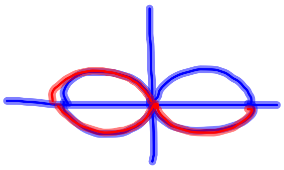
$2\sqrt{\cos\theta}$



$r = 2\sqrt{\cos\theta}$



$r = -2\sqrt{\cos\theta}$



$r^2 = 4\cos\theta$

$r^2 = 4\cos(-\theta) = 4\cos\theta$

x-axis ✓

$r^2 = 4\cos(\pi - \theta)$

$\cos\theta < 0$ ← $r^2 = -4\cos\theta$ No.

$(-r)^2 = 4\cos(-\theta)$

$r^2 = 4\cos\theta$ Yes

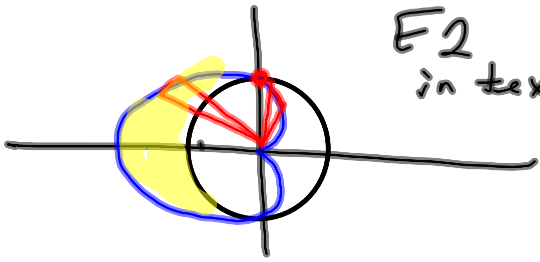
y-axis ✓

∴ origin.

Area Differential

$dA = \frac{1}{2} r^2 \, d\theta$

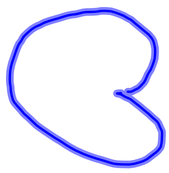
Area inside $r=1$ & outside $r = 1 - \cos\theta$



E2 in text.

$1 - \cos(\frac{\pi}{2}) = 1$

Inside Cardioid & outside $r_2 = 1$.



outer area - inner area

$$2\int_{\frac{\pi}{2}}^{\pi} \left(\frac{1}{2}(\underbrace{1-\cos\theta}_{r_1})^2 - \frac{1}{2}(\underbrace{1}_{r_2})^2\right) d\theta$$

$$= \int_{\frac{\pi}{2}}^{\pi} (1 - 2\cos\theta + \cos^2\theta - 1) \, d\theta$$

OR:

$$2\int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}(1-\cos\theta)^2 \, d\theta - \frac{\pi}{2}$$

area of $\frac{1}{2}$ circle.

Arc Length in Polar

$$L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$$

$$= \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Use $x = r\cos\theta = f(\theta)\cos\theta$

$\boxed{y = r\sin\theta}$

& prove this formula!

#29

$$\left(\frac{dx}{d\theta}\right)^2 = \left(f'(\theta)\cos\theta - f(\theta)\sin\theta\right)^2$$

$$= f'^2\cos^2\theta - 2f'f\cos\theta\sin\theta + f^2\sin^2\theta$$