$$\int \sqrt{1+\frac{1}{u}}\,du = \int \sqrt{1+\frac{1}{x}}\,dx = I$$

$$u = \left(1+\frac{1}{x}\right)^{\frac{1}{2}} \quad du = \frac{1}{2}\left(1+\frac{1}{x}\right)^{-\frac{1}{2}}\left(-\frac{1}{x^2}\right)dx \quad \Longrightarrow$$

$$dv = dx \qquad v = x$$

$$I = x\left(1+\frac{1}{x}\right)^{\frac{1}{2}} + \int \frac{1}{2}\cdot\frac{1}{x}\left(1+\frac{1}{x}\right)^{-\frac{1}{2}}dx$$

$$\int \frac{x^2\,dx}{\sqrt{x^2-1}} = \frac{1}{2}\int \frac{x\cdot 2x\,dx}{\sqrt{x^2-1}} = I$$

$x = \sec\theta$

$$u = x \qquad du = dx$$

$$dv = (x^2-1)^{-\frac{1}{2}}(2x)\,dx$$

$$v = 2\sqrt{x^2-1}$$

$$\Rightarrow I = 2x\sqrt{x^2-1} - \int 2\sqrt{x^2-1}\,dx$$

u

dv

ideas

Change #4a to $\int \frac{x^3\,dx}{\sqrt{x^2-1}}$

$$\int \frac{dx}{\sqrt{5x^2-17}}$$

$$\sqrt{5x^2-17} = \sqrt{17\left(\frac{5}{17}x^2-1\right)}$$

Let $\sec\theta = \sqrt{\frac{5}{17}}\,x$

$$= \sqrt{17(\sec^2\theta-1)} = \sqrt{17}\sqrt{\sec^2\theta-1}$$

$$\int 2\pi y \, ds \quad \text{about } x\text{-axis}$$

$$\int 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx \qquad \int 2\pi\, y\sqrt{1+g'(y)^2}\,dy$$

$x = g(y)$ is 1-to-1.

Surface area for parametrics:

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

About x-axis ($y \geq 0$): $S = \int 2\pi y\, ds$ where $x = f(t)$, $y = g(t)$

$$= \int 2\pi\, g(t)\sqrt{f'(t)^2 + g'(t)^2}\,dt$$

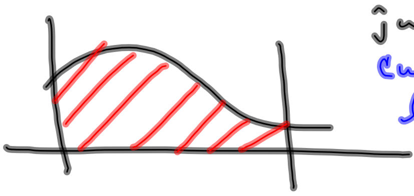
About y-axis:

$$S = \int 2\pi x\, ds = \int 2\pi f(t)\sqrt{f'(t)^2 + g'(t)^2}\,dt$$

Area : $\displaystyle\int_a^b y\,dx = \int g(t)\,f'(t)\,dt$

curve must be traced just once.

curve must pass vertical line test



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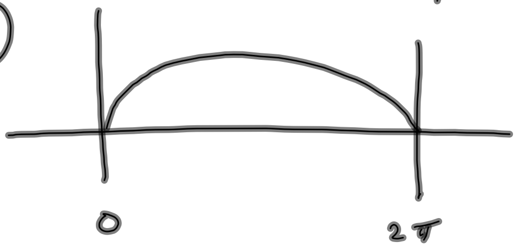
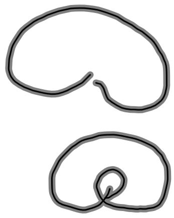




area under cycloid

$x = a(t - \sin t)\ ,\ y = a(1 - \cos t)$

$dx = a(1 - \cos t)$

$$\text{Area} = a^2 \int_0^{2\pi} (1 - \cos t)^2\,dt \quad = \int_a^b y\,dx$$

$$= a^2 \int_0^{2\pi} (1 - 2\cos t + \cos^2 t)\,dt$$

$$= a^2 \int_0^{2\pi} \left(1 - 2\cos t + \tfrac{1}{2}(1 + \cos(2t)\right)dt$$

$$= a^2 \int_0^{2\pi} \left(\tfrac{3}{2} - 2\cos t + \tfrac{1}{2}\cos(2t)\right)dt$$

$$= a^2 \left[\tfrac{3}{2}t - 2\sin t + \tfrac{1}{4}\sin(2t)\right]_0^{2\pi}$$

$$= 3a^2\pi$$

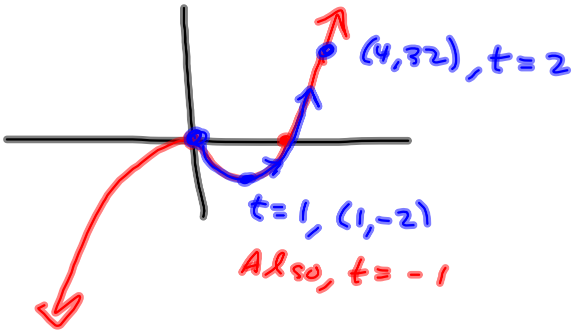
Apr 18-12:23 PM

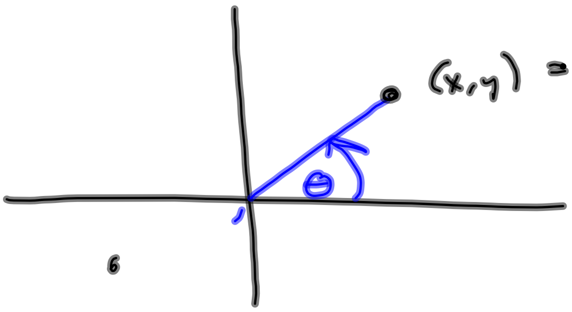
$x = t^2, \ y = t^6 - 2t^4 \qquad t \in \mathbb{R}$

$t = \pm\sqrt{x} \qquad y = (\pm\sqrt{x})^6 - 2(\pm\sqrt{x})^4$

$= x^3 - 2x^2 = x^2(x-2)$

t	x	y
0	0	0
1	1	-1
2	4	32
-1	1	-1



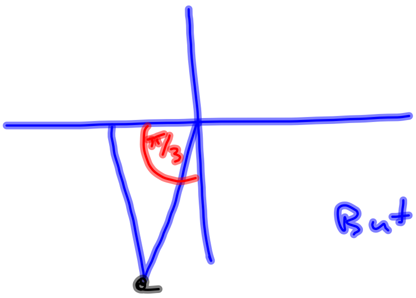


$(x,y) = (r, \theta) = \left(\sqrt{x^2+y^2}, \tan^{-1}\left(\frac{y}{x}\right)\right)$

$\tan\theta = \frac{y}{x}$

$\tan^{-1}\left(\frac{y}{x}\right)$ is only partially satisfactory.

$\mathcal{R}(\arctan *) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



But $\arctan\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$

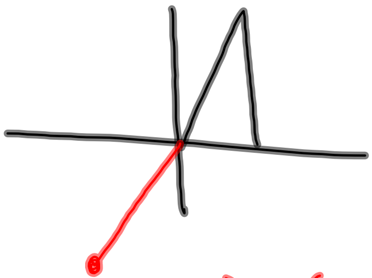
$(-1, -\sqrt{3}) = \left(2, \frac{4\pi}{3}\right) =$

$r = 2$

$\theta = \frac{4\pi}{3} = \frac{\pi}{3} + \pi$ for pts in Q $\underline{\text{III}}$

Representation is <u>NOT</u> unique!

$\left(2, \frac{4\pi}{3}\right) = \left(-2, \frac{\pi}{3}\right)$



$(-1, -\sqrt{3}) = \left(-2, \frac{\pi}{3}\right) = \left(2, \frac{4\pi}{3}\right)$

$x = r\cos\theta \qquad dx = -r\sin\theta\, d\theta$

$y = r\sin\theta \qquad dy = r\cos\theta\, d\theta$