§11.2 $\quad y = f(x) = f(x(t))$

$$\frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt} \implies \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = y'$$

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg}\cdot\frac{dg}{dx}$$

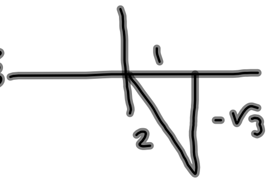
$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[y'\right] = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}}$$

Tan. Line: $x = \sin(2\pi t),\ y = \cos(2\pi t)$.

Find eq'n of tan. line to the curve @ $t = -\frac{1}{6}$.

$$\left.\frac{dy}{dx}\right|_{t=-1/6} = \left.\frac{dy/dt}{dx/dt}\right|_{t=-1/6} = \left.\frac{-2\pi \sin(2\pi t)}{2\pi \cos(2\pi t)}\right|_{t=-1/6} = \left.-\tan(2\pi t)\right|_{t=-1/6}$$

$$= -\tan\left(-\frac{\pi}{3}\right) = -(-\sqrt{3}) = \sqrt{3} = m$$

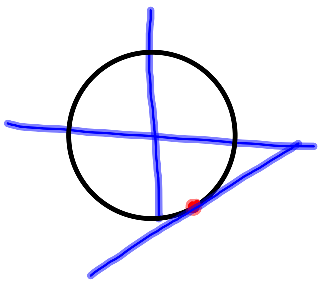


$$x\left(-\frac{1}{6}\right) = \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$y\left(-\frac{1}{6}\right) = \cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$$

$$y = m(x - x_1) + y_1$$

$$\boxed{y = \sqrt{3}\left(x + \frac{\sqrt{3}}{2}\right) + \frac{1}{2}}$$



Apr 17-12:49 PM

$$\frac{dx}{dt} = 2\pi \cos(2\pi t)$$

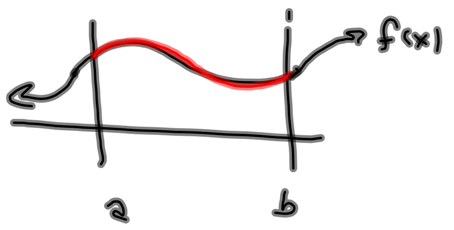
$$\frac{dy}{dx} = -\tan(2\pi t) = y'$$

$$\frac{d^2y}{dx^2} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}} = \frac{-2\pi \sec^2(2\pi t)}{2\pi \cos(2\pi t)} = -\sec^3(2\pi t)$$

Recall §6.4

$$s = \text{arc length} = \int_a^b \sqrt{\left(\frac{dx}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_a^b ds$$



$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$s(t) = \int_a^t ds = \int_a^t \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$ is an increasing function of t.

§ $x = f(t)$ & $y = g(t)$ $\longrightarrow$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$s = \int_{t=a}^{t=b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$t + \sin t$

§ 4.2 #25 $x = \cos t$, $y = t + \sin t$ $0 \le t \le \pi$

$x' = -\sin t$, $y' = 1 + \cos t$

$$x'^2 + y'^2 = \sin^2 t + 1 + 2\cos t + \cos^2 t$$

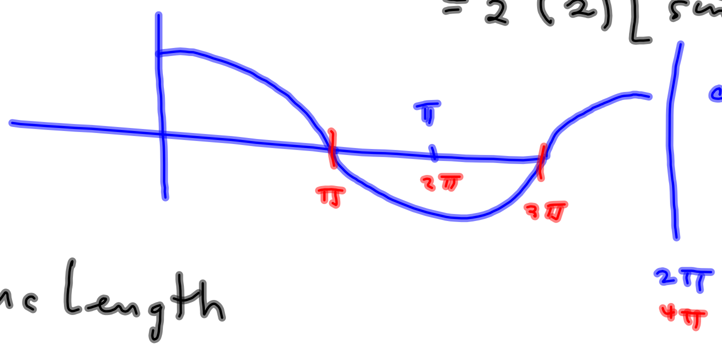
$$= 2 + 2\cos t \qquad \cos^2 t = \frac{1 + \cos(2t)}{2}$$

$$= 2(1 + \cos t) = 2\left(2\,\frac{(1 + \cos t)}{2}\right) = 4\cos^2\left(\tfrac{1}{2}t\right)$$

$$S = \sqrt{2}\int_0^{\pi} \sqrt{1+\cos t}\; dt = \int_0^{\pi} \sqrt{4\cos^2\left(\tfrac{1}{2}t\right)}\; dt$$

$$= \int_0^{\pi} 2\left|\cos\left(\tfrac{1}{2}t\right)\right| dt = 2\int_0^{\pi} \cos\left(\tfrac{1}{2}t\right) dt$$

$$= 2\,(2)\left[\sin\left(\tfrac{1}{2}t\right)\right]_0^{\pi} = 4$$



Arc Length

NOT 6.4, But 6.3

Pp 326 - 330

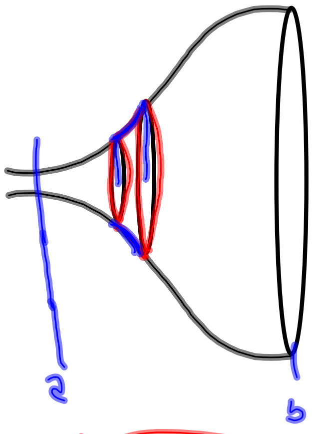
Area of Surface of Revolution is §6.4

§6.4 Pp 332-4

S = surface area = $\int_a^b 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

for revolving about the x-axis

Came from formula for frustum of a cone, See Calc I!



$$\boxed{S = \int_a^b 2\pi y \, ds} \; *$$

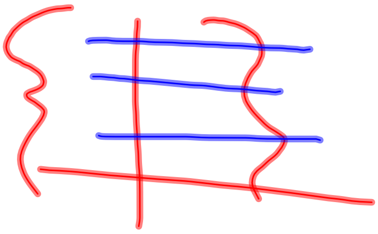
what if $x=g(y)$, here? *

$$\boxed{S = \int_a^b 2\pi y \sqrt{\left(\frac{dx}{dy}\right)^2+1}\, dy}$$

Need $g(y)$ 1-to-1

if $y=f(x)$: *

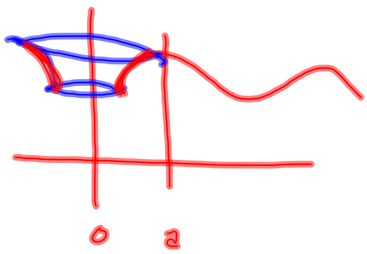
$$\boxed{S = \int 2\pi f(x) \sqrt{1+(f'(x))^2}\, dx}$$



about the y-axis:

$x=g(y)$

$$\int_a^b 2\pi \underbrace{x}_{g(y)} \sqrt{\left(\frac{dx}{dy}\right)^2+1}\; dy$$



$y=f(x)$

$$\boxed{\int_a^b 2\pi\, x \sqrt{1+(f'(x))^2}\, dx}$$

Need $f(x)$ 1-to-1 to revolve around y-axis.