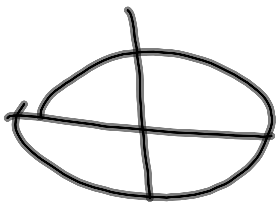
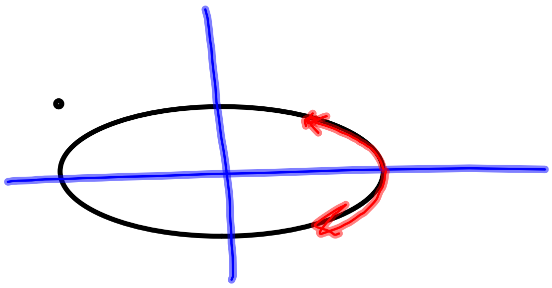
4/26 8pm Kress Theatre

Parametrization for ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$x = a\cos t$, $y = b\sin t$





10.7 #36

$$\sum_{n=1}^{\infty}\left(\sqrt{n+1}-\sqrt{n}\right)(x-3)^n = \sum_{n=1}^{\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}(x-3)^n$$

$2 < x < 4$. check endpoints:

@ $x=2$: $\sum \frac{1}{\sqrt{n+1}+\sqrt{n}}(-1)^n$ converges, by alt. series test, $a_n \longrightarrow 0$, a_n decreases

@ $x=4$: $\sum \frac{1}{\sqrt{n+1}+\sqrt{n}}$

$$\frac{d}{dn}\left[\sqrt{n+1}-\sqrt{n}\right] = \frac{1}{2}(n+1)^{-\frac{1}{2}} - \frac{1}{2}n^{-\frac{1}{2}}$$

$$= \frac{1}{2\sqrt{n+1}} - \frac{1}{2\sqrt{n}} < 0 \quad \forall\, n \geq 1$$

$$= \frac{\sqrt{n}-\sqrt{n+1}}{2\sqrt{n}\sqrt{n+1}} < 0 \quad \text{see?}$$

§10.8 #10 erratum

10 cont'd

$f^{(3)}(x) = -\frac{3}{8}(1-x)^{-\frac{5}{2}} \quad f^{(3)}(0) = -\frac{3}{8}$

$P_3(x) = -\frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3$ STOP!

$P_0(x) = 1$

$P_1(x) = 1 - \frac{1}{2}x$

$P_2(x) = 1 - \frac{1}{2}x - \frac{1}{8}x^2$

$P_3(x) = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3$

Good Student catch on teacher error.

§10.9 #33

33 $e^{\sin x} = \sum_{k=0}^{\infty} \frac{1}{k!}(\sin x)^k$

$$= \boxed{1 + \sin x + \frac{1}{2!}\sin^2 x + \frac{1}{3!}\sin^3 x + \frac{1}{4!}\sin^4 x + \cdots}$$

$$= 1 + \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\right) + \frac{1}{2!}\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\right)^2$$

$$\left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5\right)\left(x - \frac{1}{6}x^3 + \frac{1}{120}x^5\right)$$

$$= \left(x - \frac{1}{6}x^3\right)^2 + 2\left(x - \frac{1}{6}x^3\right)\left(\frac{1}{120}x^5\right) + \left(\frac{1}{120}x^5\right)^2$$

$$= x^2 - \frac{1}{3}x^4 + \frac{1}{36}x^6 + 2\left(\frac{1}{120}x^6 - \cancel{\frac{1}{720}x^8}\right) + \cancel{\frac{1}{14400}x^{10}}$$

So we only need to run it out to the 3rd power

$$\boxed{x^0,\ x,\ x^3,\ x^2}$$

Not done. Need to stop @ x^3 power, as you'll have x^0 thru x^3 represented. This makes the problem more manageable.

Apr 16-12:49 PM

§10.10 #14

14 $\left(1-\frac{x}{2}\right)^4 = 1+4\left(-\frac{x}{2}\right)+6\left(-\frac{x}{2}\right)^2+4\left(-\frac{x}{2}\right)^3+\left(-\frac{x}{2}\right)^4$

$= 1-2x+\frac{6}{4}x^2-\frac{4}{8}x^3+\frac{x^4}{24}$

$= \boxed{1-2x+\frac{3}{2}x^2-\frac{1}{2}x^3+\frac{1}{24}x^4}$

$\frac{1}{2^4}x^4 = \frac{1}{16}x^4$

Morphed the 2^4 into a 24

Apr 16-12:35 PM

§ 10.10 #25 Teacher did a MARVELOUS #26

25 $\int_0^x \sin(t^2)\,dt$ on $[0,1]$ want error $< 10^{-3}$

$$= \int_0^x \sum_{k=0}^{\infty} \frac{(-1)^k (t^2)^{2k+1}}{(2k+1)!}\,dt = \int_0^x \sum_{k=0}^{\infty} \frac{(-1)^k t^{4k+2}}{(2k+1)!}\,dt$$

$$= \left[\sum_{k=0}^{\infty} \frac{(-1)^k t^{4k+3}}{(4k+3)(2k+1)!} \right]_0^x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{4k+3}}{(4k+3)(2k+1)!}$$

$$= \frac{x^3}{3} - \frac{x^7}{7\cdot 3!} + \frac{x^{11}}{11\cdot 5!} - \frac{x^{15}}{15\cdot 7!} + \cdots$$

Want to be w/in 10^{-3}, so use alternating series test, using $x=1$, as biggest.

$\frac{1}{3}, \frac{1}{42}, \frac{1}{1320} < \frac{1}{1000}$ so

$F(x) = \frac{x^3}{3} - \frac{x^7}{42}$ does it.

#26 is a good test question.