$$\frac{\text{How many RIGHT}}{\text{How many Probs}}$$ each assignment.

Any you didn't do, let's see some work!

$f(x) = x^4 - 5x^3 + 4x^2 + 2x - 1$ @ $a = 1$

Find taylor series

$f(1) = 1 - 5 + 4 + 2 - 1 = 1$

$f'(x) = 4x^3 - 15x^2 + 8x + 2$

$f'(1) = 4 - 15 + 8 + 2 = -1$

$f''(x) = 12x^2 - 30x + 8$

$f''(1) = 12 - 30 + 8 = -10$

$f'''(x) = 24x - 30$

$f^{(3)}(1) = 24 - 30 = -6$

$f^{(4)}(x) = 24$

$f^{(5)}(x) = 0 = f^{(n)}(x) \quad \forall\, n \geq 5$

So $\cancel{f(x) = 1 - x - \frac{10}{2!}x^2 - \frac{6}{3!}x^3 + \frac{24}{4!}x^4}$ No. $a = 1$, idiot.

$f(x) = 1 - (x-1) - 5(x-1)^2 - (x-1)^3 + (x-1)^4$

This is NOT $f(x-1)$

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Possible App would be to make hand calculations easier for values close to $x=100$

E.g. $f(x) = x^4 - 5x^3 + 4x^2 + 2x - 1$

Expand about $x=100$

$$f(x) = -1 + a_1(x-100) + a_2(x-100)^2 + a_3(x-100)^3 + a_4(x-100)^4$$

This would make the powers of $x-100$ small & manageable.

hand $99^4 \longmapsto (-1)^4$ easy

Taylor's FORMULA for $P_3(x)$ on $[0,3]$

$$P_3(x) = 1 - (x-1) - 5(x-1)^2 - (x-1)^3$$

$$f^{(4)}(x) = 24$$

$$\text{So } R_3(x) = \frac{f^{(4)}(c)}{4!}(x-1)^4$$

$$= \frac{24}{4!}(x-1)^4 = (x-1)^4$$

$R_2(x)$: $P_2(x) = 1 - (x-1) - 5(x-1)^2$

$$R_2(x) = \frac{f^{(3)}(c)}{3!}(x-1)^3 \quad \text{for some } c \in [0,3]$$

Apply it: Get an estimate for the error of $P_2(x)$ on $[0,3]$

$$|R_2(x)| \le \frac{\text{MAX}\{|f^{(3)}(x)|\}}{3!}(x-1)^3$$

$f^{(3)}(x) = 24x - 30$

$$\max_{[0,3]}\{f^{(3)}(x)\} = 24(3) - 30 = 42$$

So, $|R_2(x)| \le \frac{42}{6}|x-1|^3 = 7|x-1|^3$

over the WHOLE interval,

error is $\le 7|3-1|^3 = 56$

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How about $P_4(x)$ for $\sin x$ on $[0, 3\pi]$?

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\boxed{P_4(x) = x - \frac{x^3}{3!}}$$

$f^{(1)}(x) = \cos x$

$f^{(2)}(x) = -\sin x$

$f^{(3)}(x) = -\cos x$

$f^{(4)}(x) = \sin x$

$f^{(5)}(x) = \cos x$

$$|R_4(x)| \le \frac{\max_{[0,3\pi]}\{|\cos x|\}}{5!} |x|^5$$

$$\le \frac{1}{5!}|x|^5 \le \frac{1}{120}(3\pi)^5$$

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10.5 #10 $\sum \frac{4^n}{(3n)^n}$

$$\sqrt[n]{|a_n|} = \frac{4}{3n} \xrightarrow{n\to\infty} 0$$

$$\left| \frac{4^{n+1}}{(3(n+1))^{n+1}} \cdot \frac{(3n)^n}{4^n} \right| = \frac{4 \cdot 3^n n^n}{3^{n+1}(n+1)^{n+1}}$$

$$= \frac{4}{3} \cdot \frac{n^n}{(n(1+\frac{1}{n}))^{n+1}} = \frac{4}{3} \cdot \frac{n^n}{n^{n+1}(1+\frac{1}{n})^{n+1}}$$

$$= \frac{4}{3n(1+\frac{1}{n})^{n+1}}$$

$$\left(1+\frac{1}{n}\right)^{n+1} \xrightarrow{n\to\infty}$$

$$y = \left(1+\frac{1}{n}\right)^n \left(1+\frac{1}{n}\right)^1$$

$$y = \left(1+\frac{1}{n}\right)^n$$

$$\ln y = \underset{\infty}{n} \cdot \underset{0}{\ln\left(1+\frac{1}{n}\right)} = \frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}} \xrightarrow[=]{L'H \; n\to\infty} \frac{\frac{-\frac{1}{n^2}}{1+\frac{1}{n}}}{-\frac{1}{n^2}}$$

$$= \frac{1}{1+\frac{1}{n}} \xrightarrow{n\to\infty} 1$$

$$\ln y = 1$$

$$y = e^1$$

$$\left(1+\frac{2}{n}\right)^n = \left(\left(1+\frac{1}{\left(\frac{n}{2}\right)}\right)^{\frac{n}{2}}\right)^2$$

$$\xrightarrow{n\to\infty} e^2$$