Tomorrow : For 20% of C10 Test, grade your own homework off my solutions.

Derivatives of arctan x.

$fp := \mathrm{D}(f)$

$$x \to \frac{1}{1+x^2}$$

$fp2 := \mathrm{D}(fp)$

$$x \to -\frac{2x}{\left(1+x^2\right)^2}$$

$fp3 := \mathrm{D}(fp2)$

$$x \to \frac{8x^2}{\left(1+x^2\right)^3} - \frac{2}{\left(1+x^2\right)^2}$$

$fp4 := \mathrm{D}(fp3)$

$$x \to -\frac{48x^3}{\left(1+x^2\right)^4} + \frac{24x}{\left(1+x^2\right)^3}$$

$fp5 := \mathrm{D}(fp4)$

$$x \to \frac{384x^4}{\left(1+x^2\right)^5} - \frac{288x^2}{\left(1+x^2\right)^4} + \frac{24}{\left(1+x^2\right)^3}$$

$fp5 := \mathrm{D}(fp4)$

$$x \rightarrow \frac{384\,x^4}{\left(1+x^2\right)^5} - \frac{288\,x^2}{\left(1+x^2\right)^4} + \frac{24}{\left(1+x^2\right)^3}$$

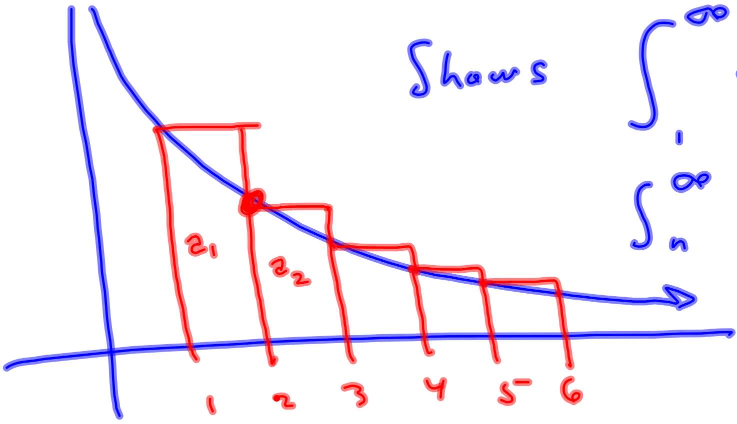
$fp6 := \mathrm{D}(fp5)$

$$x \rightarrow -\frac{3840\,x^5}{\left(1+x^2\right)^6} + \frac{3840\,x^3}{\left(1+x^2\right)^5} - \frac{720\,x}{\left(1+x^2\right)^4}$$

$fp7 := \mathrm{D}(fp6)$

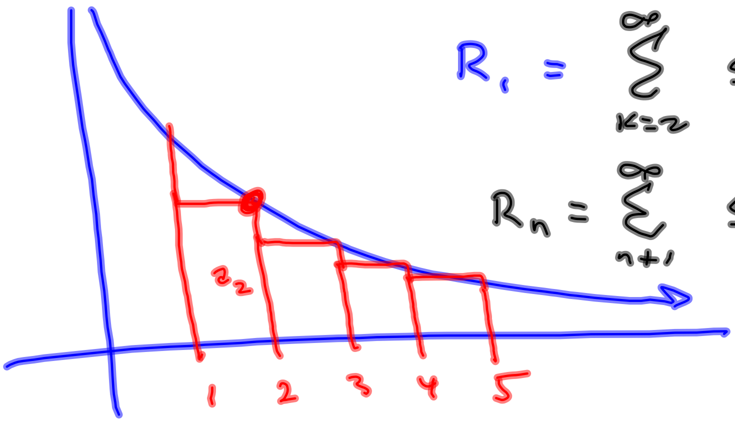
$$x \rightarrow \frac{46080\,x^6}{\left(1+x^2\right)^7} - \frac{57600\,x^4}{\left(1+x^2\right)^6} + \frac{17280\,x^2}{\left(1+x^2\right)^5} - \frac{720}{\left(1+x^2\right)^4}$$

C8 take-home Monday



Shows $\int_1^\infty \le \sum_{k=1}^\infty a_k$

$\int_n^\infty \le \sum_n^\infty$



$R_1 = \sum_{k=2}^\infty \le \int_1^\infty$

$R_n = \sum_{n+1}^\infty \le \int_n^\infty$

a_1

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10.7 # 10

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{\sqrt{n}}$$

$\left|\frac{\sim}{\sqrt{n}}\right| < |\sim|$

so, if $\sum \sim$ converges, so will $\sum \frac{\sim}{\sqrt{n}}$

Intuition / Big Picture.

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(x-1)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{(x-1)^n}\right|$$

$$= \frac{\sqrt{n}}{\sqrt{n+1}}|x-1| \xrightarrow{n\to\infty} |x-1| \overset{\text{want}}{<} 1 \longrightarrow$$

$-1 < x-1 < 1$

$0 < x < 2$

$\boxed{R = 1}$ $\left(\frac{2-0}{2} = 1\right)$

$x=0$ $\sum \frac{(0-1)^n}{\sqrt{n}} = \sum (-1)^n \cdot \frac{1}{\sqrt{n}}$ converges

$\frac{1}{\sqrt{n}}$ dec., $\frac{1}{\sqrt{n}} \xrightarrow{n\to\infty} 0$

$x=2$ $\sum \frac{(2-1)^n}{\sqrt{n}} = \sum \frac{1}{\sqrt{n}}$ diverges, p-test.

n increases $\Rightarrow \sqrt{n}$ increases

$\Rightarrow \frac{1}{\sqrt{n}}$ decreases.

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$$\frac{d}{dn}\left[\frac{1}{\sqrt{n}}\right] = \frac{d}{dn}\left[n^{-\frac{1}{2}}\right] = -\frac{1}{2}n^{-\frac{3}{2}} < 0$$

for positive n.

$$\sum_{n=2}^{\infty} \frac{x^n}{n \ln n}$$

$$\sum \frac{1}{n \ln n}$$

Also, see 10.7 #s 29, 30 and 10.3 #s 55, 54

Integral Test is great here

$$\sum \frac{1}{n(\ln n)^{\frac{57}{56}}}$$

$$\int_2^{\infty} \frac{dx}{x \ln x}$$

$$= \int_2^{\infty} \frac{1}{\ln x} \cdot \frac{1}{x}\,dx = \int_2^{\infty} (\ln x)^{-1} \left(\frac{1}{x}\,dx\right)$$

$$= \lim_{b\to\infty} \int_{\ln 2}^{b} u^{-1}\,du = \lim_{b\to\infty} \left[\ln u\right]_{\ln 2}^{b}$$

$u = \ln x$ $\qquad x = 2 \to u = \ln 2$

$du = \frac{1}{x}\,dx$ $\qquad$ " $x = \infty \to u = \ln\infty = \infty$ "

$$= \left(\lim_{b\to\infty} \ln b\right) - \ln 2 = \infty$$

$$= \lim_{b\to\infty} \left[\ln(\ln x)\right]_2^b$$

$$\int_0^1 (x^2-5x+7)^{15}(2x-5)\,dx$$

OK

$$= \left.\frac{(x^2-5x+7)^{16}}{16}\right]_0^1$$

BAD

$$\boxed{\text{NOT LEGAL: } \int_0^1 u^{15}\,du = \left.\frac{u^{16}}{16}\right]_0^1}$$

$u = x^2 - 5x + 7$

$x = 0 \rightsquigarrow u = 7$

$x = 1 \rightsquigarrow u = 3$

OK

$$\int_7^3 u^{15}\,du = \left[\frac{u^{16}}{16}\right]_7^3$$

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