arctan$(3x^4)$ in 2 ways.

Error Estimates

① Integral Test version

② Alternating Series version

$$\frac{1}{1-\cancel{x}} = 1+x+x^2+\cdots$$

① Taylor's way

② $\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2} = \boxed{(1+x^2)^{-1}}$

1^{st} 4 terms.

$$\frac{1}{1-(\cancel{-x^2})}$$

Sin x by cos x

Taylor's way

$f^{(0)}(x) = \cos x \qquad \cos(0)=1$

$f^{(1)}(x) = -\sin x \qquad -\sin(0)=0$

$f^{(2)}(x) = -\cos x \qquad -\cos(0)=-1$

$f^{(3)}(x) = \sin x \qquad \sin(0)=0$

$f^{(4)}(x) = \cos x \qquad \cos(0)=1$

$f^{(5)}(x) = -\sin x \qquad -\sin(0)=0$

$f^{(6)}(x) = -\cos x \qquad -\cos(0)=-1$

$$1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \cdots$$

$= \cos x \longrightarrow$

$$\sin x = x - \frac{1}{3}\cdot\frac{1}{2!}x^3 + \frac{1}{5}\cdot\frac{1}{4!}x^5 - \frac{1}{7}\cdot\frac{1}{6!}x^7 + \cdots$$

Using integral test to find error.
Only works for non-alternating series.

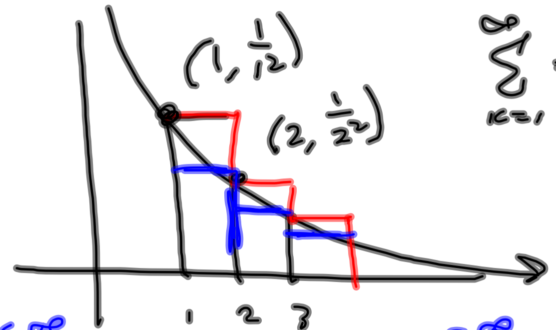
For an alternating series that converges ABSOLUTELY, the integral test would give you a VERY conservative (high) estimate for the error.

$\boxed{E}$ Estimate (Find an upper bound on) the error in using the 1^{st} 4 terms of

$\sum_{k=1}^{\infty} \frac{1}{k^2}$ Bound on $\sum_{k=5}^{\infty} \frac{1}{k^2} = R_4$

Recall

$$\boxed{\int_{5}^{\infty} \leq R_4} \leq \int_{4}^{\infty}$$

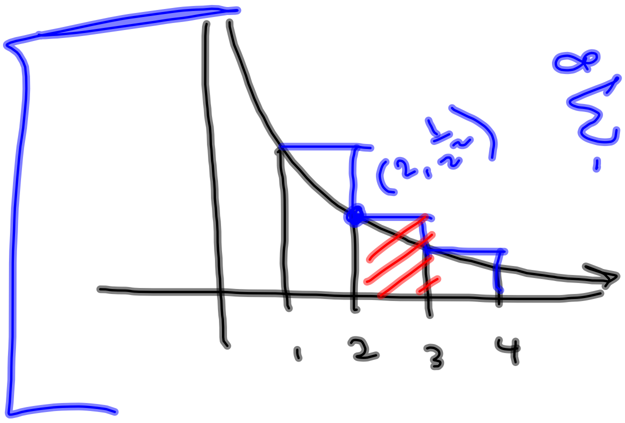


$\sum_{k=1}^{\infty}$ = area in the rectangles.

Note $\sum_{k=2}^{\infty} \leq \int_{1}^{\infty}$

$\frac{1}{x^2}$ $\sum_{k=3}^{\infty} = R_2 \leq \int_{2}^{\infty}$

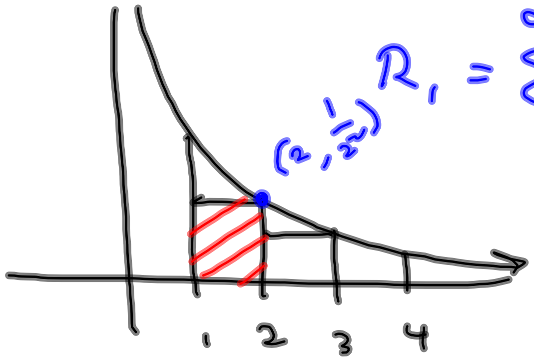
$\int_{4}^{\infty} \leq \sum_{2}^{\infty} = R_3 \leq \int_{3}^{\infty}$



$\sum_1^\infty \geq \int_1^\infty$

In general,

$\int_{n+1}^\infty \leq R_n \leq \int_n^\infty$



$R_1 = \sum_2^\infty \leq \int_1^\infty$

$\sum_2^\infty$ throw out the 1st term.

If you use RIGHT endpoints, you see how the sum fits UNDER the curve.

Integral estimate for $1^{\underline{st}}$ 4 terms

is $\int_4^\infty \geq R_4 = \sum_5^\infty$

$R_4 = \sum_{k=1}^{\infty} - \sum_{k=1}^{4} a_k x^k$ OR

$\sum_{k=0}^{\infty} - \sum_{k=0}^{4} a_k x^k$

Double-check text, but I THINK R_4 refers to $\sum_5 \cdots x^k$

In other situations, $1^{\underline{st}}$ 4 terms means $1^{\underline{st}}$ 4 nonzero terms

(For sine & cosine this takes you to 7^{th} & $6^{\underline{th}}$ powers, respectively,

$$\sum_{k=1}^{\infty} \frac{1}{k^2}$$

Error for the 1st 4 terms

$$S = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$$

$$= \boxed{\sum_{k=1}^{4} \frac{1}{k^2}} + \sum_{k=5}^{\infty} \frac{1}{k^2} \to R_4$$

$$R_4 \le \int_4^{\infty} \frac{dx}{x^2} = \lim_{b\to\infty} \int_4^b x^{-2}\,dx$$

$$= \lim_{b\to\infty} \left[-x^{-1}\right]_4^b = \lim_{b\to\infty} \left[-\frac{1}{x}\right]_4^b$$

$$= \lim_{b\to\infty} \left(-\frac{1}{b} - \left(-\frac{1}{4}\right)\right)$$

$$= \boxed{\frac{1}{4} \ge R_4}$$

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$$\sum_{k=1}^{\infty} \frac{1}{k^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{25} + \cdots$$

Alternating version

$$\sum_{k=1}^{\infty} (-1)^k \frac{1}{k^2}$$

What's estimate of the error from 1st 4 terms?

$$R_4 \le |a_5| = \frac{1}{25}$$

Integral test can only handle cases where convergence is ABSOLUTE

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$\boxed{\sum \frac{1}{n}}$

$\boxed{\sum (-1)^k \frac{1}{k}}$

$\int_{25}^{\infty} \frac{dx}{x}$ diverges