Term-by-term product §10.7 ~~#~~ 51
will get 1st 6 terms.
One piece of it! the x^{11} coefficient.

$$-\left[\frac{1}{10!}+\frac{1}{3!8!}+\frac{1}{5!6!}+\frac{1}{7!4!}+\frac{1}{9!2!}+\frac{1}{11!}\right]x^{11}$$

$$\binom{11}{3}=\binom{11}{8}$$

11! is LCD.

$$=-\left[\frac{10+165+462+330+55+1}{11!}\right]x^{11}$$

$$\frac{11!}{3!8!}=\frac{11\cdot \cancel{10}^{5}\cdot \cancel{9}^{3}}{\cancel{3}\cdot\cancel{2}}=11\cdot 15=165$$

$$\frac{11!}{5!6!}=\frac{11\cdot\cancel{10}^{2}\cdot\cancel{9}^{3}\cdot\cancel{8}\cdot 7}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}}=462$$

$$\frac{11!}{7!4!}=\frac{11\cdot 10\cdot \cancel{9}^{3}\cdot 8}{4\cdot 3\cdot 2}=330$$

$$\frac{11!}{9!2!}=\frac{11\cdot 10}{2}=55$$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

1 10 45 120 210 252 210 120 45 10 1

1 11 55 165 330 462 462 . . .

$\frac{11!}{10!\,1!}$ $\frac{11!}{9!\,2!}$ $\frac{11!}{8!\,3!}$ $\frac{11!}{7!\,4!}$ $\frac{11!}{6!\,5!}$

10.7 $\qquad R, S, I$

$\sum_{n=0}^{\infty} (e^x - 4)^n \qquad$ Geometric, $r = e^x - 4$, $a = 1$

Need $|r| = |e^x - 4| < 1$

$-1 < e^x - 4 < 1$

$3 < e^x < 5$

$\{x \mid \ln 3 < x < \ln 5\} = (\ln 3, \ln 5) = I$

$S = \frac{1}{1-(e^x-4)} = \boxed{\frac{1}{5-e^x} = S}$

$\boxed{R = \frac{\ln 5 - \ln 3}{2}} = \frac{1}{2}\ln\left(\frac{5}{3}\right) = \ln\left(\sqrt{1.\overline{6}}\right)$

$\binom{m}{0} = 1$, $\binom{m}{1} = m$, $\binom{m}{2} = \frac{m(m-1)}{2!}$ Not sure why they need this.

$$\boxed{\binom{m}{k} = \frac{m(m-1)\cdots(m-k+1)}{k!}}$$

Old-fashioned formula:

$$\binom{m}{k} = \frac{m!}{(m-k)!\,k!}$$

when $m \in \mathbb{N}$ works great.

$$\frac{m!}{1!(m-1)!} = m$$

$$\binom{m}{0} = \frac{m!}{m!\,0!} = 1$$

works better for m = fraction.

$$\binom{\frac{1}{2}}{0} = \frac{\left(\frac{1}{2}\right)!}{\left(\left(\frac{1}{2}\right)-0\right)!\,0!} = 1$$

, but we have NO idea what $\left(\frac{1}{2}\right)!$ looks like.

$$\binom{\frac{1}{2}}{1} = \frac{\left(\frac{1}{2}\right)!}{\left(\frac{1}{2}-1\right)!\,1!}$$

Breaks down, here, I'm confused about $\left(\frac{1}{2}\right)!$ & $\left(-\frac{3}{2}\right)!$

10.10 #10 1st 4 terms.

$$\frac{x}{\sqrt[3]{1+x}} = x\left(\frac{1}{(1+x)^{1/3}}\right) = x(1+x)^{-\frac{1}{3}}$$

$$(1+x)^{-\frac{1}{3}} = 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{36}{3!\,3^3}x^3 + \cdots$$

$$\binom{-\frac{1}{3}}{2} = \frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)}{2!} = \frac{1}{2}\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right) = \frac{2}{3^2}$$

$$\binom{-\frac{1}{3}}{3} = \frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3!} = \frac{-\left(\frac{1\cdot 4\cdot 7}{3^3}\right)}{3!}$$

$$= \frac{36}{3!\,3^3}$$

$$\text{So } \frac{x}{\sqrt[3]{1+x}} = \boxed{x - \frac{1}{3}x^2 + \frac{2}{9}x^3 - \frac{36}{3!\,3^3}x^4} + \cdots$$

$$\binom{-\frac{1}{3}}{4} = \frac{1\cdot 4\cdot 7\cdot 10}{4!\,3^4}$$

Apr 8-12:36 PM

$$S - S_n = R_n$$

Integral Test.
n^{th} term (alternating)
Taylor's Remainder Results
→ Only works on Absolutely convergent series.

$$\sum a_n(x-a)^n = S = S_n + R_n = S_n + \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$$

$$\& \left| \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1} \right| \leq \left| \frac{M}{(n+1)!}(x-a)^{n+1} \right|$$

where $M \geq \max\{|f^{(n+1)}(t)| \mid t \text{ between } x \,\&\, a\}$

x will be given, or an interval will be given.

for $|x-a|$ when interval is given, just take endpoint for x.