§10.10 #s 3, 8, 11, 14, 15, 17, 21, 25

(26) $F(x) = \int_0^x t^2 e^{-t^2} dt = -\frac{1}{2}\int_0^x t\cdot(-2te^{-t^2}dt)$

$u = t$, $\boxed{dv = -2te^{-t^2}dt}$

$du = 1$ $v = e^{-t^2}$

$uv - \int v\,du = -\frac{1}{2}\left[te^{-t^2} - \boxed{\int e^{-t^2}dt}\right]$

Goes in circles, imho.

Not an elementary form. (Closed-form)

$F(x) = \int_0^x f(t)\,dt$

$f(x) = \left[1 + (-x^2) + \frac{1}{2!}(-x^2)^2 + \frac{1}{3!}(-x^2)^3 + \dots\right]x^2$

$= x^2\left[1 - x^2 + \frac{1}{2!}x^4 - \frac{1}{3!}x^6 + \dots\right]$

$= x^2 - x^4 + \frac{1}{2!}x^6 - \frac{1}{3!}x^8 + \dots$

So, $\int_0^x f(t)\,dt =$

$\left[\frac{1}{3}t^3 - \frac{1}{5}t^5 + \frac{1}{7}\cdot\frac{1}{2!}\cdot t^7 - \frac{1}{9}\cdot\frac{1}{3!}t^9 - \dots\right]_0^1$

$= \frac{1}{3}x^3 - \frac{1}{5}x^5 + \frac{1}{7}\cdot\frac{1}{2!}x^7 - \frac{1}{9}\cdot\frac{1}{3!}x^9 + \cdots$

want this to represent $F(x)$ on $[0,1]$ to within .001 of actual.

$|R_n(x)| \le |a_{n+1}|$

$\left|\frac{1}{3}x^3\right| \le \frac{1}{3}$ on $[0,1]$

$\frac{1}{5}$ max

$\frac{1}{14}$ max

$\frac{1}{54}$

$\frac{1}{11}\cdot\frac{1}{4!} \approx$

$$\frac{1}{11}\cdot\frac{1}{4!}$$

$$\frac{1}{264}$$

evalf(%)

$$0.003787878788$$

$$\frac{1}{13}\cdot\frac{1}{5!}$$

$$\frac{1}{1560}$$

evalf(%)

$$0.0006410256410$$

So, we run it out to the 11^{th} power, since the 13^{th} power term < error tolerance.

$F(x) \approx$

$$F(x) \approx \frac{1}{3}x^3 - \frac{1}{5}x^5 + \frac{1}{7}\cdot\frac{1}{2!}x^7 - \frac{1}{9}\cdot\frac{1}{3!}x^9 + \frac{1}{11}\cdot\frac{1}{4!}x^{11}$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}\left(\sqrt{n+1}-\sqrt{n}\right)$$

$a_n \geq 0$

$\boxed{a_n \text{ decreasing}}$?

$$a_n = \sqrt{n+1} - \sqrt{n}$$

$$\frac{d}{dx}\left[(x+1)^{\frac{1}{2}} - x^{\frac{1}{2}}\right]$$

$$= \frac{\sqrt{n+1}-\sqrt{n}}{1}\cdot\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}$$

$$= \frac{1}{2}(x+1)^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}}$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{x+1}} - \frac{1}{\sqrt{x}}\right] < 0 \quad \forall x > 0$$

$$= \frac{n+1-n}{\sqrt{n+1}+\sqrt{n}}$$

$$= \frac{1}{\sqrt{n+1}+\sqrt{n}}$$

by inspection, decreases as n increases which is quicker than what I did with the derivative.

$\xrightarrow{n\to\infty} 0$ so conditionally convergent.

Absolute convergence?

means $\sum |a_n|$ converges

$$= \sum\left(\sqrt{n+1}-\sqrt{n}\right) = \boxed{\sum \frac{1}{\sqrt{n+1}+\sqrt{n}}}$$

I did this

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\sqrt{n+2}-\sqrt{n+1}}{\sqrt{n+1}-\sqrt{n}} = \frac{\sqrt{n}\left[\sqrt{1+\frac{2}{n}} - \sqrt{1+\frac{1}{n}}\right]}{\sqrt{n}\left[\sqrt{1+\frac{1}{n}} - 1\right]}$$

$$\xrightarrow{n\to\infty} \frac{1-1}{1-1} \;!?$$

Waste of time!

Intuition let me down.
Intuition should've been Jon's:

p-test, $p = \frac{1}{2}$

$\sum b_n = \sum \frac{1}{\sqrt{n}}$

$$\frac{a_n}{b_n} = \frac{\frac{1}{\sqrt{n+1}+\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \frac{\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} = \frac{\sqrt{n}}{\sqrt{n}\left[\sqrt{1+\frac{1}{n}}+1\right]}$$

$$\xrightarrow{n\to\infty} \frac{1}{1+1} = \frac{1}{2}$$

$$\sqrt{n+1} = \sqrt{n\left(1+\frac{1}{n}\right)} = \sqrt{n}\sqrt{1+\frac{1}{n}}$$

$$= \left(n\left(1+\frac{1}{n}\right)\right)^{\frac{1}{2}} = n^{\frac{1}{2}}\left(1+\frac{1}{n}\right)^{\frac{1}{2}}$$

So $\sum a_n$ diverges, also.

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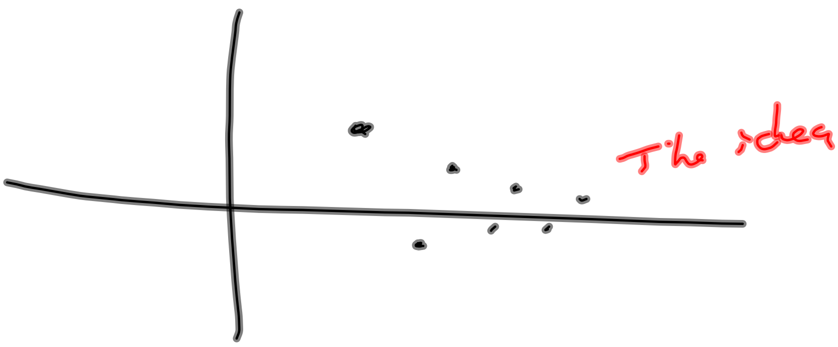
$$\cancel{f(n) = \begin{cases} \frac{1}{2^n} & \text{if } n \text{ is even} \\ \frac{1}{2^{n-1}-1} & \text{if } n \text{ is odd} \end{cases}}$$

Feeble



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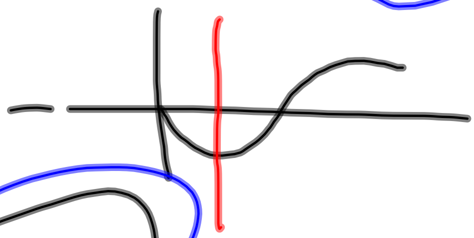
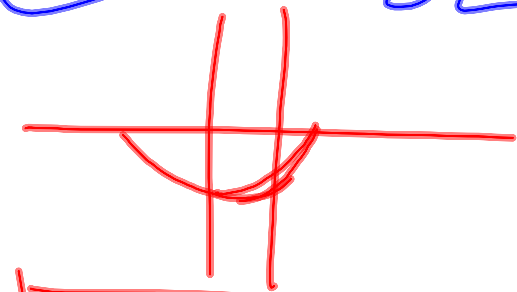
Error $= R_2(x) = \frac{f^{(3)}(c)}{3!}|x|^3 \le \frac{\max f^{(3)}(x)}{3!}|.5|^3$





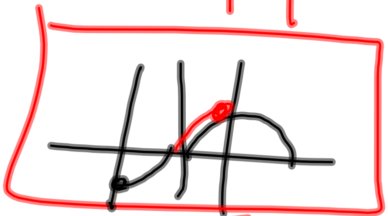
sin(.5)

cos

- sin

- cos

sin



0.5

$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{f^{(3)}(c)}{3!}x^3$$

for some $c \in [-.5, .5]$

c between x & 0=a