$\mathbb{N} = \{1, 2, 3, \ldots\}$

$\binom{m}{k} = \frac{m!}{(m-k)!\,k!} \quad \forall\, m \in \mathbb{N}$

$\binom{m}{k} = \frac{m(m-1)\cdots\boxed{(m-k+1)}}{k!}$ for any m

$k \geq 3$

$\binom{m}{1} = m$, $\binom{m}{2} = \frac{m(m-1)}{2!}$ fits the formula

$m-1 = \boxed{m-2+1}$

$$\sqrt[3]{1+x} = (1+x)^{\frac{1}{3}}$$

$$= 1 + \sum_{k=1}^{\infty} \binom{\frac{1}{3}}{k} x^k$$

$$\binom{\frac{1}{3}}{1} = \frac{1}{3}$$

$$\left(\frac{1}{3}\left(-2\right)\right)$$

$$\binom{\frac{1}{3}}{2} = \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!} = \frac{-\frac{1}{3}\cdot\frac{2}{3}}{2!}$$

$$\binom{\frac{1}{3}}{3} = \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3!} = \frac{\frac{1}{3}\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)}{3!}$$

$$= \frac{\frac{1(2)(5)}{3^3}}{3!} = \frac{2(5)}{3^3 3!}$$

$$\binom{\frac{1}{3}}{4} = \frac{2(5)\left(\frac{1}{3}-3\right)}{3^3\cdot 4!} = -\frac{2\cdot 5\cdot \frac{8}{3}}{3^3\cdot 4!} = \frac{-2\cdot 5\cdot 8}{3^4\cdot 4!}$$

$$\binom{\frac{1}{3}}{5} = \frac{2\cdot 5\cdot 8\cdot 11}{3^5\cdot 5!} \longrightarrow \frac{1}{3} - 4 = \frac{-11}{3}$$

$$2\cdot 5\cdot \cdots (3(n-1)-1)$$

$$1 + \frac{1}{3}x - \frac{2}{2!3^2}x^2 + \frac{2\cdot5}{3!3^3}x^3 - \frac{2\cdot5\cdot8}{4!3^4}x^4 + \dots + (-1)^{n+1}\frac{2\cdot5\cdots(3n-4)}{n!\cdot3^n}x^n + \dots$$

$$= (1+x)^{\frac{1}{3}}$$

$$2\cdot5\cdot(3n-3-1)$$
$$= 2\cdot5\cdots(3n-4)$$

$$\Rightarrow (1+x^2)^{\frac{1}{3}}$$

$$= 1 + \frac{1}{3}x^2 - \frac{2}{2!3^2}(x^2)^2 + \frac{2\cdot5}{3!3^3}(x^2)^3 - \frac{2\cdot5\cdot8}{4!3^4}(x^2)^4 + \dots$$

$$= 1 + \frac{1}{3}x^2 - \frac{2}{2!3^2}x^4 + \boxed{\frac{2\cdot5}{3!3^3}x^6} - \frac{2\cdot5\cdot8}{4!3^4}x^8 + \dots$$

\# 18 $\int$ 10.10 Approximate to within 0.001

$$\int_0^{.25} \boxed{\sqrt[3]{1+x^2}}\,dx = \int_0^{.25}\left[1 + \frac{1}{3}x^2 - \frac{1}{3^2}x^4 + \dots\right]dx$$

$$= \left[x + \frac{1}{3}\cdot\frac{1}{3}x^3 - \frac{1}{5}\cdot\frac{1}{3^2}x^5 + \frac{1}{7}\cdot\frac{2\cdot 5}{3!\,3^3}x^7 + \cdots \right]_0^{.25}$$

$$= \frac{1}{4} + \frac{1}{3}\cdot\frac{1}{3}\left(\frac{1}{4}\right)^3 - \frac{1}{5}\cdot\frac{1}{3^2}\left(\frac{1}{4}\right)^5 + \frac{1}{7}\cdot\frac{2\cdot 5}{3!\,3^3}\left(\frac{1}{4}\right)^7 + \cdots$$

$\frac{1}{4} = .25$

$\frac{1}{3}\cdot\frac{1}{3}\cdot\left(\frac{1}{4}\right)^3 \approx .001736$ Nawp

$\frac{1}{5}\cdot\frac{1}{3^2}\left(\frac{1}{4}\right)^5 \approx .0000217$ Yep,

cut it off @ the term before :

$\approx \frac{1}{4} + \frac{1}{3}\cdot\frac{1}{3}\left(\frac{1}{4}\right)^3$

$$\frac{1}{3^2\cdot 4^3}$$

$$\frac{1}{576}$$

$evalf(\%)$

$$0.001736111111$$

$$\frac{1}{5\cdot 3^2\cdot 4^5}$$

$$\frac{1}{46080}$$

$evalf(\%)$

$$0.00002170138889$$

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$$\frac{1}{4}+\frac{1}{3^2\cdot 4^3}$$

$$\frac{145}{576}$$

evalf(%)

$$0.2517361111$$

$$\int_0^{.25}\left(1+x^2\right)^{\frac{1}{3}}\,dx$$

Looks like we nailed it.

$$0.2517149311$$

#16 $\int_0^{0.2}\frac{e^{-x}-1}{x}\,dx$ Improper; w/in .001

$$\frac{e^{-x}-1}{x}=-\frac{1}{x}+\frac{e^{-x}}{x}=-\frac{1}{x}+\frac{1-x+\frac{x^2}{2}-\frac{x^3}{3!}+\cdots}{x}$$

$$=-\frac{1}{x}+\frac{1}{x}-1+\frac{x}{2}-\frac{x^2}{3!}+\frac{x^3}{4!}\ \cdots$$

So, $\int_0^{.2}\frac{e^{-x}-1}{x}\,dx=\left[-x+\frac{1}{2}\cdot\frac{x^2}{2}-\frac{1}{3}\cdot\frac{x^3}{3!}+\frac{1}{4}\cdot\frac{x^4}{4!}+\cdots\right]_0^{.2}$

I think you can stop @ 2nd term

$$\sum_{k=1}^{5} \frac{(-1)^k \cdot x^k}{k \cdot k!}$$

$$-x + \frac{1}{4}x^2 - \frac{1}{18}x^3 + \frac{1}{96}x^4 - \frac{1}{600}x^5$$

$$f := x \rightarrow \sum_{k=1}^{2} \frac{(-1)^k \cdot x^k}{k \cdot k!}$$

$$x \rightarrow \sum_{k=1}^{2} \frac{(-1)^k x^k}{k\, k!}$$

$f(.2)$

$$-0.1900000000$$

Only needed two terms.

#26 $F(x) = \int_0^x t^2 e^{-t^2}\,dt$

Approximate w/in 0.001 on [0, 1]

⋮

$F(x) = \left[\frac{1}{3}x^3 - \frac{1}{5}x^5 + \frac{1}{7}\cdot\frac{1}{2}x^7 - \frac{1}{9}\cdot\frac{1}{3!}x^9 + \ldots\right]$

$\frac{1}{3}$

$\frac{1}{5}$

$\frac{1}{7}\cdot\frac{1}{2}$

$\frac{1}{54}$

$\frac{1}{11}\cdot\frac{1}{4!}$