#33 $e^{\sin x}$

1st 4 nonzero terms.
0, 1, 2, 3 is 1st thought
The x^3's cancel
Pushes us to 4th degree

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

Binomial Coefficients

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

$$e^{\sin x} = 1 + \sin x + \frac{1}{2!}\sin^2 x + \frac{1}{3!}\sin^3 x + \frac{1}{4!}\sin^4 x + \cdots$$

$$\sin x = \boxed{x - \frac{1}{3!}x^3} + \quad + \cdots$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = \boxed{a^3} + \underline{3a^2b} + 3ab^2 + b^3$$

Salvaging only up to x^4 terms.

① $\sin^2 x = \left(x - \frac{1}{3!}x^3 + \cdots\right)^2$

② $\sin^3 x = \left(x - \frac{1}{3!}x^3 + \cdots\right)^3$

③ $\sin^4 x = \left(x - \frac{1}{3!}x^3 + \cdots\right)^4$

① $x^2 - \frac{2}{3!}x^3 \cdot x + \cancel{\left(\frac{1}{3!}x^3\right)^2}$

② $x^3 - \cancel{3 \cdot x^2 \frac{2}{3!} x^3}$

③ $x^4 - \cancel{4 \cdot x^3 \cdot \frac{1}{3!} x^3}$

x^4

$\sin^5 x = x^5 + \cdots \quad -$

So

$$e^{\sin x} = 1 + \left(x - \frac{1}{3!}x^3\right) + \frac{\left(x^2 - \frac{2}{3!}x^4\right)}{2!} + \frac{1}{3!}\left(x^3\right) + \frac{x^4}{4!}$$

$$= 1 + x - \frac{1}{3!}x^3 + \frac{1}{2!}\left(x^2 - \frac{2}{3!}x^4\right) + \frac{1}{3!}x^3 + \frac{1}{4!}x^4$$

$$\frac{3}{4\cdot 3\cdot 2\cdot 1} = \frac{1}{8}$$

$$= 1 + x + \frac{1}{2}x^2 - \frac{1}{3!}x^4 + \frac{1}{4!}x^4$$

$$= 1 + x + \frac{1}{2}x^2 + \frac{-4+1}{4!}x^4 = 1 + x + \frac{1}{2}x^2 - \frac{1}{8}x^4$$

$$(x - x^3)^4 = \boxed{x^4} - 4x^3\cdot x^3 + 6x^2(x^3)^2 - 4x(x^3)^3 + (x^3)^4$$

§10.10

$$(1+x)^m = \sum_{k=0}^{\infty} \binom{m}{k} x^k \text{, where } \binom{m}{k} = \frac{m!}{k!(m-k)!}$$

(if $m > 0$ $m \in \mathbb{Z}$)

If $m > 0$ $m \in \mathbb{Z}$, then this TERMINATES @ $k = m$

$$= \frac{m\cdot(m-1)\cdot(m-2)\cdots(m-k+1)(m-k)!}{k!\,(m-k)!}$$

$$= \boxed{\frac{m(m-1)(m-2)\cdots(m-k+1)}{k!} = \binom{m}{k}}$$

More general: works for ANY m.

$$\binom{7}{4} = \frac{7!}{4!3!} = \frac{7\cdot6\cdot5}{3!} = 35$$

$$\binom{7}{3} = \frac{7!}{3!\,4!} = 35 \qquad \binom{m}{k} = \binom{m}{m-k}$$

$_nP_r$ Choose & arrange $= \dfrac{n!}{(n-r)!}$

$_nC_r$ Choose $= \dfrac{n!}{r!\,(n-r)!}$

$\dfrac{3!}{0!} = 6$ $\{a, b, c\}$

$_7P_4$

$_7P_3 = \dfrac{7!}{4!} = 7 \cdot 6 \cdot 5$

$_7C_3 = \dfrac{7!}{3!\,4!}$

abc
acb
bac
bca
cab
cba

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$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & \binom{1}{0} & & \binom{1}{1} & & & \\ & & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & & \\ & \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} & \\ \binom{4}{0} & & \binom{4}{1} & & \binom{4}{2} & & \binom{4}{3} & & \binom{4}{4} \end{array}$$

$$\boxed{\binom{m}{k} = \frac{m(m-1)(m-2)\cdots(m-k+1)}{k!}}$$

Also works for $m \notin \mathbb{Z}$ or $m < 0$

$$(1+x)^7 = 1 + 7x + \binom{7}{2}x^2 + \binom{7}{3}x^3 + \binom{7}{4}x^4 + \binom{7}{5}x^5 + \binom{7}{6}x^6 + \binom{7}{7}x^7$$

$(1+x)^{-1}$

$\binom{-1}{0} = \frac{-1(0)}{0!}$

-1

$-1-1$

$k=0$

$k+1=1$

$m-k+1$

$m-0+1$

$=m+1$

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Pg 597 $(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \dots$

$+ \binom{m}{k} x^k + \dots$

$= 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k$

I had issues w/ using the formula for the $1^{\underline{st}}$ term.

$(1+x)^{-1} = \frac{1}{1-(-x)} = 1 - x + x^2 - x^3 + \dots$

$\binom{-1}{k} = \frac{-1(-2)(-3)\cdots(-1-k+1)}{k!}$

$= \frac{-1(-2)\cdots(-k)}{k!} = \frac{(-1)^k k!}{k!} = (-1)^k$

$\frac{1}{(1+x)^2} = 1 +$ $\quad -\frac{1}{1+x}$

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$\binom{-2}{1} = \frac{-2}{1}$

$K = 1$

$K - 1 = 0$

$-2-(k-1) = -2-0 = -2-k+1$

$m-k+1$

$m-(k-1)$

$\binom{-2}{2} = \frac{-2(-2-(1))}{2!} = \frac{-2(-3)}{2!}$

$= -2-(-2-(2-1))$

$\binom{-2}{3} = \frac{-2(-2-1)(-2-2)}{3!}$

$k-1 = 3-1 = 2$

$\binom{-2}{4} = \frac{-2(-2-1)(-2-2)(-2-3)}{4!}$

$= \frac{-2(-3)(-4)(-5)}{4!}$

$\binom{-2}{k}$

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