$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$$

$$e^{ix} = 1 + ix - \frac{x^2}{2} - \frac{i}{3!}x^3 + \frac{x^4}{4!} + i\frac{x^5}{5!} - \frac{x^6}{6!} - i\frac{x^7}{7!} + \cdots$$

$$e^{-ix} = 1 - ix - \frac{x^2}{2} + \frac{i}{3!}x^3 + \frac{x^4}{4!} - i\frac{x^5}{5!} - \frac{x^6}{6} + i\frac{x^7}{7!} + \cdots$$

$$e^{ix} + e^{-ix} = 2 - 2\frac{x^2}{2} + 2\frac{x^4}{4!} - 2\frac{x^6}{6!} + \cdots -$$

$$= 2\left[1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\right]$$

$$= 2\cos x \Longrightarrow$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

$$\sin x = \frac{e^{ix} - e^{ix}}{2}$$

Apr 2-12:09 PM

#s 1-10 Taylor's by Substitution

6 $\cos\left(\frac{x^{2/3}}{\sqrt{2}}\right)$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots$$

$$\cos\left(\frac{x^{2/3}}{\sqrt{2}}\right) = 1 - \frac{1}{2}\cdot\left(\frac{x^{2/3}}{2^{1/2}}\right)^2 + \frac{1}{4!}\left(\frac{x^{2/3}}{2^{1/2}}\right)^4 +$$

$$\cdots + \frac{(-1)^n}{(2n)!}\left(\frac{x^{2/3}}{2^{1/2}}\right)^{2n} + \cdots$$

$(2n)(2/3) = \frac{4n}{3}$

$(\frac{1}{2})(2n) = n$

$$= 1 - \frac{1}{2}\frac{x^{4/3}}{2^1} + \frac{1}{4!}\frac{x^{8/3}}{2^2} + \cdots + \frac{(-1)^n}{(2n)!}\frac{x^{\frac{4n}{3}}}{2^n} + \cdots$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}\frac{x^{4k/3}}{2^k}$$

#s 11-28 Power Series Ops

(12) $x^2 \sin x = x^2\left[x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots\right]$

$= x^3 - \frac{x^5}{3!} + \frac{x^7}{5!} + \cdots + (-1)^n \frac{x^{2n+3}}{(2n+1)!} + \cdots$

$x^{2n+1} \cdot x^2 = x^{2n+1+2} = x^{2n+3}$

#s 29-34 1^{st} 4 nonzero terms.

(30) $\frac{\ln(1+x)}{1-x} = \ln(1+x) \cdot \frac{1}{1-x}$

$\int \frac{dx}{1+x} = \int (1 - x + x^2 - x^3 + x^4 + \cdots) dx = \int \frac{dx}{1-(-x)} = \int \frac{d}{dx}[\ln(1+x)]$

$= \ln(1+x) = \left[x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + \cdots\right] + C$

$\ln(1+0) = \ln 1 = 0 = C$

$\boxed{1^{st} \text{ 4 terms}}$

So, $\ln(1+x) \cdot \frac{1}{1-x} =$

$\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + \cdots\right)\left(1 + x + x^2 + x^3 + x^4 + x^5 + \cdots\right)$

x, x^2, x^3, x^4 So 1^{st} 4 terms is:

$$\left(x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\right)\left(1+x+x^2+x^3\right)=$$

$$x+x^2+x^3+x^4+$$
$$-\frac{1}{2}x^2-\frac{1}{2}x^3-\frac{1}{2}x^4$$
$$\frac{1}{3}x^3+\frac{1}{3}x^4$$
$$-\frac{1}{4}x^4$$

$$x(1+x+x^2+x^3)+$$
$$-\frac{1}{2}x^2(1+x+x^2)+$$
$$\frac{1}{3}x^3(1+x)+$$
$$-\frac{1}{4}x^4(1)$$

$$=x+\frac{1}{2}x^2+\left(1-\frac{1}{2}+\frac{1}{3}\right)x^3+\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\right)x^4$$

$$=x+\frac{1}{2}x^2+\frac{5}{6}x^3+\frac{7}{12}x^4$$

$$\frac{d}{dx}\left[\arctan x\right] = \frac{1}{1+x^2}$$

$$\int \frac{dx}{1+x^2} = \arctan x + C$$

$$\frac{1}{1+x^2} = \frac{1}{1-\boxed{(-x^2)}} =$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

$$= 1 + x + x^2 + x^3 + \cdots$$

$$\frac{d}{dx}\left[\ln(1+x)\right] = \frac{1}{1+x} = \frac{1}{1-\boxed{(-x)}}$$

#35 $R_3(0) = 0$

$x - \frac{x^3}{3!}$

36 $f(x) = e^x$, $P_4(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!}$

Error for $e^{\frac{1}{2}}$, using $P_4(x)$ is?

$a = 0$
$x = \frac{1}{2}$

$\frac{M}{(n+1)!} |x-a|^{n+1}$

$M \geq f^{(n+1)}(x)$ on $[0, \frac{1}{2}]$ e^x on $[-4, 4]$

$f^{(5)}(x) = e^x$ increasing.

$$\max_{[0,\frac{1}{2}]} \{ |f^{(5)}(x)| \} = e^{\frac{1}{2}}$$

$$R_4\left(\tfrac{1}{2}\right) \le \frac{e^{\frac{1}{2}}}{5!} \left|\tfrac{1}{2} - 0\right|^5 = \frac{e^{\frac{1}{2}}}{5!}\left(\frac{1}{2^5}\right)$$

$\approx .000429354497$

$$e^{\frac{1}{2}} \cdot \frac{1}{2^5} \cdot \frac{1}{5!}$$

$$\frac{1}{3840} e^{\frac{1}{2}}$$

$evalf(\%)$

0.0004293544977

$$p4 := x \rightarrow \sum_{k=0}^{4} \frac{x^k}{k!}$$

$$x \rightarrow \sum_{k=0}^{4} \frac{x^k}{k!}$$

$$p4\left(\frac{1}{2}\right) - e^{\frac{1}{2}}$$

$$\frac{211}{128} - e^{\frac{1}{2}}$$

$evalf(\%)$

−0.000283771

"Actual" error is less than .0004 3

10.9 II 4, 22, 33, 38

Yesterday's probs: Bonus Homework.
10.9 I

Read #s 45-52