10.4 #s 4, 10, 18, 24

#s 1-8 Direct Comparison

Diverge: find something smaller that diverges

Converge: " " bigger " converges.

④ $\sum_{n=2}^{\infty} \frac{n+2}{n^2-n} = \sum a_n$

Instinct: $b_n = \frac{1}{n}$ & $\sum \frac{1}{n}$ diverges.

$$a_n = \frac{n+2}{n^2-n} > \frac{n}{n^2-n} > \frac{n}{n^2} = \frac{1}{n} = b_n$$

$\sum b_n$ diverges, by $p=1$-test. $\Longrightarrow$

$\sum a_n$ diverges.

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#s 9-16 Limit comparison.

$\sum \frac{n^2+1}{n^2-1}$ $\sum \frac{n^2+1}{n^2+11}$ $\sum \frac{n^2-1}{n^2-11}$ $\sum \frac{n^2-1}{n^2+11}$

16 $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^2+2}} = \sum a_n$ Instinct : $\sqrt{\frac{n}{n^2}} = \sqrt{\frac{1}{n}} = \frac{1}{\sqrt{n}}$ &

$\sum b_n = \sum \frac{1}{n^{1/2}}$ diverges by $p=\frac{1}{2}$-test

$\sqrt{ab} = \sqrt{a}\sqrt{b}$

Limit Comparison:

$$\frac{a_n}{b_n} = \frac{\sqrt{\frac{n+1}{n^2+2}}}{\frac{1}{\sqrt{n}}} = \sqrt{\frac{n+1}{n^2+2}} \cdot \frac{\sqrt{n}}{1} = \sqrt{\frac{(n+1)n}{n^2+2}}$$

$$= \sqrt{\frac{n^2+n}{n^2+2}} = \sqrt{\frac{n^2(1+\frac{1}{n})}{n^2(1+\frac{2}{n^2})}} = \sqrt{\frac{1+\frac{1}{n}}{1+\frac{2}{n^2}}} \xrightarrow{n\to\infty} 1$$

$\Rightarrow \sum a_n$ diverges.

DIRECT COMPARISON IS HARDER

want

$\sqrt{\frac{n+1}{n^2+2}} <$ something

$$a_n = \sqrt{\frac{n+1}{n^2+2}} < \sqrt{\frac{n+n}{n^2}} = \sqrt{\frac{2n}{n^2}} = \sqrt{\frac{2}{n}} = b_n$$

$b_n = \sqrt{\frac{2}{n}} = \frac{\sqrt{2}}{\sqrt{n}}$ & $\sum \frac{2}{n^{1/2}}$ diverges ($p=\frac{1}{2}$-test) $\Rightarrow$

$\sum a_n$ diverges.

18 $\sum_{n=1}^{\infty} \frac{3}{n+\sqrt{n}}$ #s 17-54 Any Method. Back it up.

Instinct: $\frac{3}{n+\sqrt{n}} \approx \frac{3}{n} = b_n$ & $\sum \frac{3}{n}$ diverges, by p=1-test.

$$\frac{a_n}{b_n} = \frac{\frac{3}{n+\sqrt{n}}}{\frac{3}{n}} = \frac{3}{n+\sqrt{n}} \cdot \frac{n}{3} = \frac{n}{n(1+\frac{1}{\sqrt{n}})} = \frac{1}{1+\frac{1}{\sqrt{n}}} \xrightarrow{n \to \infty} 1$$

$\Longrightarrow$ $\sum a_n$ diverges, also.

DIRECT: Diverges. Find something smaller that diverges

$$\frac{3}{n+\sqrt{n}} > \frac{3}{n+n} = \frac{3}{2n} = b_n$$

$\sum b_n$ diverges: p = 1

$\Longrightarrow$ $\sum a_n$ diverges by DIRECT comparison

~~24~~ $\sum \frac{\ln(n)^2}{n^3}$ #28

(24) $\sum \frac{5n^3-3n}{n^2(n-2)(n^2+5)} = \sum \frac{5n^3-3n}{n^2 \cdot n \cdot n^2(1-\frac{2}{n})(1+\frac{5}{n^2})} = \sum a_n$

$= \frac{5n^3+\sim\sim}{n^5+\sim\sim} \approx \frac{1}{n^2} = b_n$ & $5\sum \frac{1}{n^2}$ converges.

$\frac{a_n}{b_n} = \frac{5n^3-3n}{n^5(1-\frac{2}{n})(1+\frac{5}{n^2})} \cdot \frac{n^2}{1} = \frac{5n^5-3n^3}{n^5(1-\frac{2}{n})(1+\frac{5}{n^2})}$

$= \frac{n^5(5-\frac{3}{n^2})}{n^5(1-\frac{2}{n})(1+\frac{5}{n^2})} \xrightarrow{n\to\infty} \frac{5}{1} = 5$

$\Rightarrow \sum a_n$ converges b/c $\sum b_n$ converges by p=2-test $\left(\sum \frac{5}{n^2}\right)$

§10.9 Recall, we did arctan($3x^4$)
by integrating, term-by-term on the
derivative of arctan(x) & finding the
constant @ the end. $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ or $\sum_{n=1}^{\infty} x^{n-1}$

$$\frac{d}{dx}\left[\arctan x\right] = \frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty} (-x^2)^n$$

$$= \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

Jon & I did it the $\sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k$

It was doable, but tedious.
Always good when you can recognize
a variation on $\frac{1}{1-x}$

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§10.9 Summary

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

$$= \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!} x^k + \sum_{k=n+1}^{\infty} \frac{f^{(k)}(0)}{k!} x^k$$

$$= S_n' + R_n$$

$$= S_n' + \frac{f^{(n+1)}(c)}{(n+1)!} x^{n+1}$$

for some c between x and 0.
(In the "centered @ a" versions, between x & a)

So, $R_n = \frac{f^{(n+1)}(c)}{(n+1)!} x^{n+1}$

Practical App!

$|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1}$, where

$M = \max\{|f^{(n+1)}(x)|\}$ on the interval

$[a,x]$ OR $[x,a]$, depending on $a<x$ OR $a>x$.

Boils down to maximizing the $(n+1)^{th}$ derivative on a closed interval.

Do sine & cosine:

1. cosine using Taylor's
2. " by differentiating sine, term-by-term
3. $\cos(3x)$ by substitution

§10.9 #s 7, 16

~~7 $f(x) = \sin x$, $x = \frac{\pi}{4}$~~