Last time

$f(3x^4) = \arctan(3x^4)$ by trickery

This time: Taylor's formula

$\arctan(0) = 0$

$$\int \frac{dx}{1+x^2} = \arctan x$$

$f'(x) = \frac{1}{1+x^2}$ $\quad f'(0) = 1$

$f''(x) = -(1+x^2)^{-2}(2x)$ $\quad f''(0) = 0$

$f^{(3)}(x) = 2(1+x^2)^{-3}(2x)^2 - (1+x^2)^{-2}(2)$

$f^{(3)}(0) = -2$

$f^{(4)}(x) = -6(1+x^2)^{-4}(2x)^3 + 2(1+x^2)^{-3}(2)(2x)(2)$

$\quad + 2(1+x^2)^{-3}(2x)(2)$

$f^{(4)}(0) = 0$

$$f^{(4)}(x) = -\frac{6(2x)^3}{(x^2+1)^4} + \frac{3\cdot 8x}{(x^2+1)^3}$$

$$= \frac{-6\cdot 2^3\cdot x^3 + 3\cdot 8x\cdot(x^2+1)}{(x^2+1)^4}$$

$$= \frac{-6\cdot 2^3\cdot x^3 + 3\cdot 2^3\cdot x^3 + 3\cdot 2^3\cdot x}{(\ )^4}$$

$$= \frac{-3\cdot 2^3 x^3 + 3\cdot 2^3 x}{(\ )^4} = \frac{3\cdot 2^3 x[x^2+1]}{(x^2+1)^4}$$

$$= \frac{3\cdot 2^3 x}{(x^2+1)^3}$$

$$f^{(5)}(x) = 3\cdot 2^3 \left[\frac{(x^2+1)^3 - x\left(3(x^2+1)^2(2x)\right)}{(\quad)^6} \right]$$

$$= 3\cdot 2^3 \left[\frac{x^2+1-6x^2}{(\quad)^4} \right] = 3\cdot 2^3 \left[\frac{-5x^2+1}{(\quad)^4} \right]$$

$$f^{(5)}(0) = 3\cdot 2^3$$

0

$$\arctan(x) = x - \frac{2}{3!}x^3 + \frac{3\cdot 2^3}{5!}x^5 + \cdots$$

$$= x - \frac{1}{3}x^3 + \frac{1}{5}x^5 + \cdots$$

From this, we get

$\arctan(3x^4)$

$$\frac{\cancel{3\cdot 2\cdot 2\cdot 2}}{5\cdot \cancel{4\cdot 3\cdot 2}}$$

$$= (3x^4) - \frac{1}{3}(3x^4)^3 + \frac{1}{5}(3x^4)^5 + \cdots$$

$$= 3x^4 - \frac{1}{3}\cdot 3^3 x^{12} + \frac{1}{5}\cdot 3^5 x^{20} + \cdots$$