§10.3

#s 1–10 use integral test. Check conditions for integral test.

#6: $\frac{1}{n(\ln n)^2} \xrightarrow{n\to\infty} 0$ ✓

$f'(n) = \frac{d}{dn}\left[\frac{1}{n(\ln n)^2}\right] = \frac{d}{dn}\left[(n(\ln n)^2)^{-1}\right]$

$= -(n(\ln n)^2)^{-2}\left[(\ln n)^2 + n(2\ln n)'(\frac{1}{n})\right] < 0$ ✓

other approach

n is increasing fn of n

ln n " " " " "

∴ $\frac{1}{n \ln n}$ is decreasing fn of n

#34 needed L'Hopital to get $\sec^2(0) = 1 \neq 0$ as punchline. Diverges.

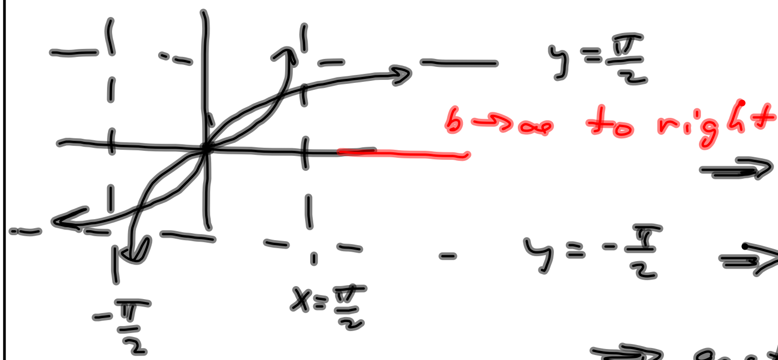
#50 want $\sum_{2}^{\infty} \frac{1}{n^2+4}$ w/in 0.1 of actual.

$$R_n \le \int_n^\infty \frac{1}{x^2+4}\,dx = \int_n^\infty \frac{1}{x^2+2^2}\,dx$$

$$= \lim_{b\to\infty} \frac{1}{2}\arctan\left(\frac{x}{2}\right)\Big]_n^b$$

$$= \lim_{b\to\infty} \frac{1}{2}\arctan\left(\frac{b}{2}\right) - \frac{1}{2}\arctan\left(\frac{n}{2}\right)$$

$$= \frac{1}{2}\cdot\frac{\pi}{2} - \frac{1}{2}\arctan\left(\frac{n}{2}\right)$$



want < 0.1

$$\Rightarrow \frac{\pi}{4} - \frac{1}{2}\arctan\left(\frac{n}{2}\right) < .1$$

$$\Rightarrow -\frac{1}{2}\arctan\left(\frac{n}{2}\right) < .1 - \frac{\pi}{4}$$

$$\Rightarrow \arctan\left(\frac{n}{2}\right) > -2\left(.1 - \frac{\pi}{4}\right)$$

$$\Rightarrow \frac{n}{2} > \tan\left(\frac{\pi}{2} - .2\right)$$

$$\Rightarrow n > 2\tan\left(\frac{\pi}{2} - .2\right)$$ Degree Mode BAD!

≈ 9.866309751

$n = 10$ does it.

$$S_{10} = \frac{1}{2^2+4} + \frac{1}{3^2+4} + \cdots + \frac{1}{10^2+4}$$

$$S_{10}' = \frac{1}{2^2+4} + \frac{1}{3^2+4} + \cdots + \frac{1}{10^2+4} \approx .3663597329$$

$$\approx .37$$

$$\sum_{n=2}^{\infty} \frac{1}{n^2+4}$$

$$-\frac{13}{40} + \frac{1}{4}\pi \coth(2\pi)$$

evalf(%)

$$0.4604036418$$

$$\sum_{n=2}^{10} \frac{1}{n^2+4}$$

$$\frac{4977759}{13587080}$$

evalf(%)

$$0.3663597329$$

§ 10.9

Taylor Series centered @ a is

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

for suitable f $\boxed{\text{(Has derivatives of all orders)}}$

& $f(b) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(b-a)^n$: f

$b \in I$ = interval of convergence for $f(x)$'s Taylor series @ a.

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

$$R_n(x) = \sum_{k=n+1}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k = (n+1)\text{-tail}$$

A number for fixed x.

$$= \frac{f^{(n+1)}(c)}{(n+1)!}\boxed{(x-a)^{n+1}} \text{ for some}$$

c between x and a.

$\leq \frac{M}{(n+1)!}$ where

$M \geq |f^{(n+1)}(x)|$

on $[a, x]$ (or $[x, a]$)

This is helpful, why?

T24 : Estimating Remainder

$$|R_n(x)| \le M \frac{|x-a|^{n+1}}{(n+1)!}, \text{ where}$$

M is a bound on $f^{(n+1)}(x)$ on

$[a,x]$ or $[x,a]$ (depending on $x > a$ or $x < a$, respectively.)

If the inequality holds, independent of n, then the series converges to $f(x)$.

Mar 28-12:45 PM

8 $\arctan(3x^4)$ $\quad a=0$

$$\left.\begin{array}{l} f(0) \\ f'(0) \\ f''(0) \end{array}\right\} \text{Tough!}$$

Suggestions:

1. Build one for $\arctan(x)$ & sub in $3x^4$ for x.

2. Look for ways to use previous results. (Term-by-term int/diff)

$\arctan x = ?$

Mar 28-12:50 PM

$$\frac{d}{dx}\left[\arctan x\right] = \frac{1}{1+x^2} = \frac{1}{1-(-x^2)}$$

$$= 1 + (-x^2) + (-x^2)^2 + \cdots + (-x^2)^n + \cdots$$

$$a_n = (-x^2)^n = (-1)^n (x^2)^n = (-1)^n x^{2n}$$

$$\left((-x)^2\right)^n = x^{2n}$$

$\therefore$ Integrate to arctan:

$$\arctan(x) = \int \sum_{n=0}^{\infty} (-1)^n x^{2n}\, dx$$

$$= \boxed{\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}} + C \longrightarrow$$ Taylor series for $\arctan x$

$$\arctan(0) = 0 = \sum 0 + \boxed{C = 0}$$

$$\therefore \arctan(3x^4) = \sum_{n=0}^{\infty} (-1)^n \frac{(3x^4)^{2n+1}}{2n+1}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{3^{2n+1} x^{8n+4}}{2n+1}$$