§10.2 Becareful with the notation

10.8 #10

$$\begin{array}{c} 32 \\ \times 8 \\ \hline 256 \end{array}$$

$a=4$

#9: $f(x) = \sqrt{x} = x^{\frac{1}{2}}$ $\qquad f(4) = 2$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$ $\qquad f'(4) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$

$f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}$ $\qquad f''(4) = -\frac{1}{4}\left(\frac{1}{2}\right)^3 = -\frac{1}{32}$

$f'''(x) = \frac{3}{8}x^{-\frac{5}{2}}$ $\qquad f'''(4) = \frac{3}{8}\left(\frac{1}{2}\right)^5 = \frac{3}{256}$

$$T_3(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{32}\frac{(x-4)^2}{2!} + \frac{3}{256}\frac{(x-4)^3}{3!}$$

$\sum \frac{f^{(k)}(a)}{k!}(x-a)^k$

$\frac{3}{256} \to f^{(k)}(a)$, $3 \to k$, $3! \to k!$

$f := x \rightarrow \sqrt{x}$

$$x \rightarrow \sqrt{x}$$

$f1 := \mathrm{D}(f)$

$$x \rightarrow \frac{1}{2\sqrt{x}}$$

$f2 := \mathrm{D}(f1)$

$$x \rightarrow -\frac{1}{4\left(\sqrt{x}\right)^3}$$

$f3 := \mathrm{D}(f2)$

$$x \rightarrow \frac{3}{8\left(\sqrt{x}\right)^5}$$

$[f(4), f1(4), f2(4), f3(4)]$

$$\left[2, \frac{1}{4}, -\frac{1}{32}, \frac{3}{256}\right]$$

$f(4) + f1(4)\cdot(x-4) + \frac{f2(4)\cdot(x-4)^2}{2} + \frac{f3(4)\cdot(x-4)^3}{3!}$

$$1 + \frac{1}{4}x - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3$$

$expand(\%)$

$$\frac{5}{8} + \frac{15}{32}x - \frac{5}{128}x^2 + \frac{1}{512}x^3$$

with(*Student*[*Calculus1*]) :
TaylorApproximation(x^(1/2), 4, '*view*'= [0 .. 8, −.283 .. 3.11], '*order*'= 3, '*output*'='*polynomial*');

$$\frac{5}{8} + \frac{15}{32}x - \frac{5}{128}x^2 + \frac{1}{512}x^3$$

$2\sin x\cos x$: $\sin x \cos x$ $=$

$$\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\frac{x^9}{9!}-\frac{x^{11}}{11!}\right)\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}-\frac{x^{10}}{10!}\right)=$$

To get 1st 6 terms, need to get to the 11th power

$$x\left[1-\frac{x^2}{2!}+\cdots-\frac{x^{10}}{10!}\right]+$$

$10!$

$$-\frac{x^3}{3!}\left[1-\frac{x^2}{2!}+\cdots+\frac{x^8}{8!}\right]+$$

$$\frac{x^5}{5!}\left[1\ \cdots\ -\frac{x^6}{6!}\right]+$$

$$\frac{x^7}{7!}$$

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$$\int\left(\sum f_k(x)\right)dx \quad + C$$

$$= \sum \int \left(f_k(x)\,dx + c_k\right), \text{ where } \sum c_k = C$$

$$\int\left(\sum \frac{x^k}{k!}\right)dx = \sum_{k=0}^{\infty} \int \frac{x^k}{k!}\,dx$$

$$= \sum_{k=0}^{\infty} \frac{x^{k+1}}{k+1}\cdot\frac{1}{k!} + C = \sum_{k=0}^{\infty} \frac{x^{k+1}}{(k+1)!} + C$$

$$= \left(\frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \right) + C \quad \text{\& we}$$

know $e^0 = (\ 0\ ) + C = 1$

This gives $1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots = e^x$