§10.1 #46

$$-\frac{1}{2^n} \le \frac{\sin^2 n}{2^n} \le \frac{1}{2^n}, \text{ b/c } -1 \le \sin^2 n \le 1$$

$n \to \infty$ $\downarrow$ $n \to \infty$ $\downarrow$

$$0 \le \lim_{n\to\infty} \frac{\sin^2 n}{2^n} \le 0$$

§10.1 #92

$$a_1 = -1, \quad a_{n+1} = \frac{a_n + 6}{a_n + 2}$$

$$a_2 = \frac{-1+6}{-1+2} = 5$$

$$a_3 = \frac{5+6}{5+2} = \frac{11}{7} \approx$$

$$a_4 = \frac{\frac{11}{7}+6}{\frac{11}{7}+2} = \frac{\frac{53}{7}}{\frac{25}{7}} = \frac{53}{25}$$

Hard to show decreasing.
Hard to show bounded below.
So it's hard to use Bounded Convergence Theorem, like I wanted. BUT instructions say Assume a_n converges to something, say, L.

Then $\lim_{n\to\infty} a_{n+1} = L$ and

$\lim_{n\to\infty} \frac{a_n+6}{a_n+2} = \frac{L+6}{L+2}$ This means

$L = \frac{L+6}{L+2} \Longrightarrow$ $L^2+2L = L+6$

$L^2+L-6=0$

$(L+3)(L-2)=0$

$L=-3$ or $L=2$

How do we know $L=-3$ isn't right?

Suppose $a_n > 0$ for some n.

Then $a_n+6>0$ & $a_n+2>0$, so

$\frac{a_n+6}{a_n+2} = a_{n+1} > 0$, too.

Since $a_2=5$, we have $a_n>0$ for all $n \geq 2$.

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$$\frac{d}{dx}\left[\frac{x^n}{n!}\right] = n\frac{x^{n-1}}{n!}$$

$$= \frac{\cancel{n}\, x^{n-1}}{\cancel{n}(n-1)(n-2)\cdots(1)} = \frac{x^{n-1}}{(n-1)!}$$

$$\int\left(\sum_{n=0}^{\infty} a_n x^n\right)dx = \left(\sum_{n=0}^{\infty} a_n \frac{x^{n+1}}{n+1}\right) + C$$

Find C by plugging in $x=0$ into

$$e^x = e^0 = 1 = \left(\sum_{n=0}^{\infty} a_n \frac{0^{n+1}}{n+1}\right) + C$$

$$\int \tan x\, dx = -\int \frac{-\sin x}{\cos x}dx = -\ln|\cos x| + C$$

$$= \ln|(\cos x)^{-1}| + C = \ln|\sec x| + C$$

10.8's about

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(2)}{n!}(x-2)^n \quad \text{From } \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

You'll use this plus any and all tricks in § 10.7.

$$\frac{1}{1-x} = \boxed{1 + x + x^2 + \cdots} \quad \text{Geometric.}$$

$$\frac{1}{1-(-x^2)} = \frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots$$

$$\sin\left(2\left(x - \frac{\pi}{6}\right)\right) = ?$$

$$\sin(-17x^{11}) =$$

$$\sin\left(2\left(x-\frac{\pi}{6}\right)\right) = 2\sin\left(x-\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)$$

$$= 2\left(\sum \frac{(-1)^n}{(2n+1)!}\left(x-\frac{\pi}{6}\right)^{2n+1}\right)\left(\sum \frac{(-1)^n}{(2n)!}\left(x-\frac{\pi}{6}\right)^{2n}\right)$$

This is $2\sin x \cos x$ shifted right $\frac{\pi}{6}$.

Coefficients obtained for $a=0$

$$\sin x = \sum_{n=0}^{\infty} \frac{f^{(n)}\left(\frac{\pi}{6}\right)}{(2n+1)!}\left(x-\frac{\pi}{6}\right)^{2n+1}$$ is expansion centered at $\frac{\pi}{6}=a$

$$\sin\left(x-\frac{\pi}{6}\right) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)\left(x-\frac{\pi}{6}\right)^{2n+1}}{(2n+1)!}$$ is expansion centered @ $x=0=a$, shifted righ $\frac{\pi}{6}$ units.