§10.7 #22

$$\left| \frac{3^{2(n+1)} (x-2)^{n+1}}{3(n+1)} \cdot \frac{3n}{3^{2n}(x-2)^n} \right|$$

$$\frac{3^{2n+2}}{3^{2n}} = 3^{2n+2-2n}$$

$$= 3^2$$

$$\frac{3n}{3n+3}$$

$$= \left| 3^2 (x-2) \frac{3n}{3n+3} \right| \xrightarrow{n \to \infty} 9|x-2| \overset{\text{want}}{<} 1$$

(i) $|x-2| < \frac{1}{9}$

$$-\frac{1}{9} < x-2 < \frac{1}{9}$$

$$\frac{17}{9} < x < \frac{19}{9}$$

$$\left(\frac{1}{9}\right)^n = \left(\frac{1}{3^2}\right)^n$$

$$= \frac{1^n}{(3^2)^n} = \frac{1}{3^{2n}}$$

0

$x = \frac{19}{9}$: $\displaystyle\sum_0^\infty \frac{(-1)^n 3^{2n}}{3n} \left(\frac{1}{9}\right)^n =$

$$= \sum \frac{(-1)^n 3^{2n} \cdot \frac{1}{3^{2n}}}{3n} = \sum (-1)^n \frac{1}{3n}$$

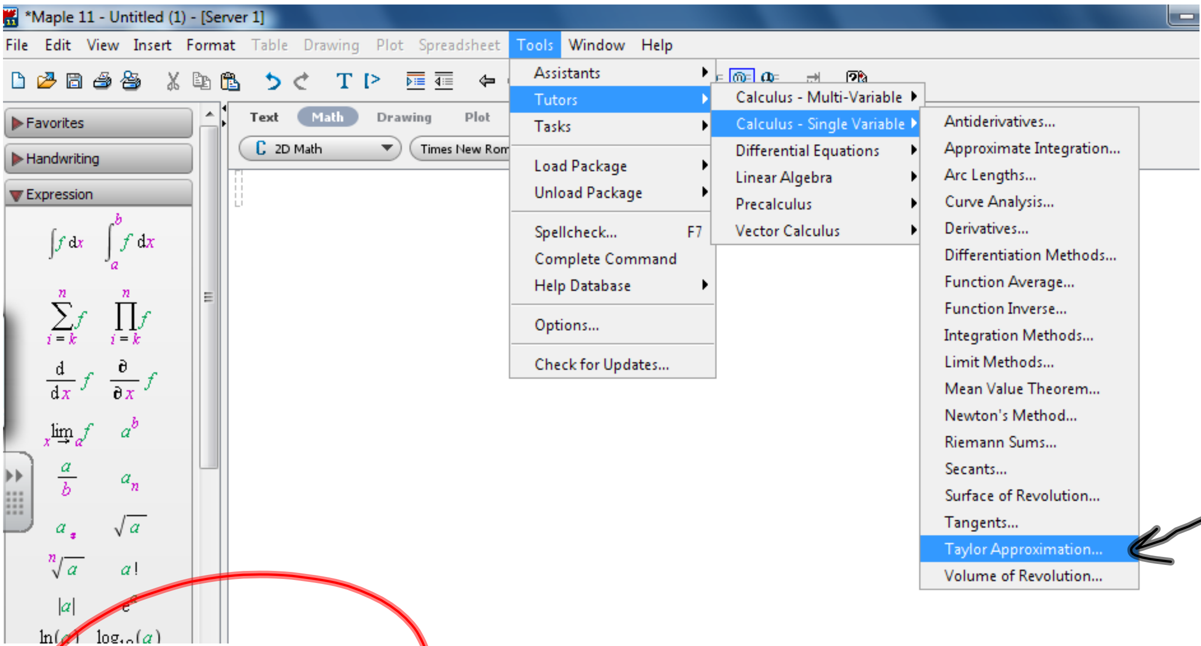
Other endpt: $\left(-\frac{1}{9}\right)^n = (-1)^n \left(\frac{1}{9}\right)^n$

$I = \left(\frac{17}{9}, \frac{19}{9}\right]$

$\to$ cancels the other $(-1)^n$:

$(-1)^n (-1)^n = (-1)^{2n} = 1$

On Monday, let's do some 10.8 probs with Maple. Taylor Approx is the tool:



Execute

with(Student[Calculus1])

(x+5)*(x+1)

x's y's

TaylorApproximation((x+5)(x+1), 0, 'view' = [-4 .. 4, -20. .. 30.], 'order' = 4, 'output' = 'polynomial')

is the command

you pick the window

or plot

or animation

$$\frac{x+5}{x+1} = 5-4x+4x^2-4x^3+4x^4$$
$$= c_0+c_1x+c_2x^2+c_3x^3+c_4x^4$$

is Taylor Polynomial of order 4, centered at $x=0$, for $f(x)=\frac{x+5}{x+1}$.

Done w/ Maple.

Recall, IF $f(x)=\sum_{n=0}^{\infty} c_n x^n$ is the power series (Taylor) expansion for $f(x)$ @ $x=0$, then $\boxed{c_n = \frac{f^{(n)}(0)}{n!}}$

Recall: $f(x) = c_0+c_1x+c_2x^2+c_3x^3+\dots$

Then $f(0) = c_0+c_1\cdot 0+c_2\cdot 0^2 = c_0 = f(0)$

$f'(x) = c_1+2c_2x+3c_3x^2+\dots$

$f'(0)=c_1$

$f''(x) = 2c_2+3\cdot 2c_3x+\dots$

$f''(0)=2c_2 \Rightarrow c_2 = \frac{f''(0)}{2}$

$f'''(x) = 3\cdot 2c_3 + 4\cdot 3\cdot 2c_4x+\dots$

$f'''(0) = 3\cdot 2c_3$

$\Rightarrow c_3 = \frac{f'''(0)}{3!}$

Example:
$$\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{k=0}^{\infty} (-1)^k x^k$$

$$= 1 - x + x^2 - x^3 + x^4 + \cdots$$

Use k's to handle

$$S_n = \sum_{k=0}^{n} f(x_k)$$

Can't use 'n' twice

10.8 method:

$f(x) = \frac{1}{x+1} = (x+1)^{-1}$ $\quad f(0) = 1$ $\quad c_0 = \frac{1}{0!} = 1$

$f'(x) = -(x+1)^{-2}$ $\quad f'(0) = -\frac{1}{(0+1)^2} = -1$ $\quad c_1 = \frac{-1}{1!} = -1$

$f''(x) = 2(x+1)^{-3}$ $\quad f''(0) = 2$ $\quad c_2 = \frac{2}{2!} = 1$

$f'''(x) = -6(x+1)^{-4}$ $\quad f'''(0) = -6$ $\quad c_3 = \frac{-6}{3!} = -1$

The other way is to recognize $\frac{1}{1+x}$ as merely a variation on $\frac{1}{1-x}$, i.e., $\frac{1}{1+x} = \frac{1}{1-(-x)}$

So replace x by $-x$ in the Geometric series

$$\sum_0^\infty x^n = \frac{1}{1-x} \quad : \quad \frac{1}{1-(-x)} = \sum_0^\infty (-x)^n =$$

$$= \sum_0^\infty ((-1)x)^n = \sum_0^\infty (-1)^n x^n = 1 - x + x^2 - x^3 + x^4 + \dots$$

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$$\frac{x+5}{x+1} = \frac{x+5}{1+x} = \frac{x+5}{1-(-x)}$$

$$= \frac{x}{1-(-x)} + \frac{5}{1-(-x)} =$$

$$= x\left(\frac{1}{1-(-x)}\right) + 5\left(\frac{1}{1-(-x)}\right)$$

$$= x\sum_{n=0}^{\infty}(-1)^n x^n + 5\sum_{n=0}^{\infty}(-1)^n x^n$$

∞

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$$= \sum_{n=0} (-1)^n x^{n+1} + \sum_{n=0}^{\infty} (-1)^n (5) x^n$$

$$= (-1)^0 x^{0+1} + (-1)^1 x^{1+1} + (-1)^2 x^{2+1} + \dots$$

$$+ 5 - 5x + 5x^2 - 5x^3 + \dots$$

$$\begin{array}{r} = 5 - 5x + 5x^2 - 5x^3 + \dots \\ + \quad x - x^2 + x^3 \\ \hline 5 - 4x + 4x^2 - 4x^3 \end{array}$$

10.8 #s 1, 4, 7, 10, 13, 14, 17, 20, 23, 26, 29, 35
HAND IN EVENS
PLAY w/ ODDS.