$\boxed{T21}$ Term-by-term differentiation.

The derivative has same Radius of convergency as the original. Still check endpoints.

App: Find power series for $\sec x \tan x$, using $\sec x$ series.

§10.~~7~~ II #s 42, 47, 51, 52, 53, 54

$$\sum (e^x - 1)^n$$

$\boxed{T22}$ Term-by-Term Integration - Same as differentiation in $\boxed{T21}$, only <u>more</u> cool apps.

Integration is harder, sometimes impossible.

$\int e^{x^2} dx$ has NO closed-form expression.

e^{-x^2} is the basis for the bell curve!

(Use some constants to make

$$K \int_{-\infty}^{\infty} e^{-x^2} dx = 1$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

μ = mean

σ = standard deviation

Impossible to use FTC II; however,

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$e^{-x^2} = 1 + (-x^2) + \frac{(-x^2)^2}{2} + \frac{(-x^2)^3}{3!} + \frac{(-x^2)^4}{4!} + \cdots$$

Easy to integrate the 1st n terms of this series.

$$\int e^{-x^2}dx = \int\left(1 - x^2 + \frac{x^4}{2} - \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots\right)dx$$

§10.7 # 49

$$1 - \frac{1}{2}(x-3) + \frac{1}{4}(x-3)^2 + \cdots + \left(-\frac{1}{2}\right)^n (x-3)^n + \cdots$$

$$= 1 - \frac{1}{2}(x-3) + \frac{1}{2^2}(x-3)^2 + \cdots + \frac{(-1)^n}{2^n}(x-3)^n + \cdots$$

$$= 1 - \left(\frac{x-3}{2}\right) + \frac{(x-3)^2}{2^2} + \cdots + (-1)^n \frac{(x-3)^n}{2^n} + \cdots +$$

$$= 1 + \left((-1)\frac{x-3}{2}\right)^1 + \left((-1)\frac{x-3}{2}\right)^2 + \cdots + \left((-1)\frac{x-3}{2}\right)^n + \cdots$$

$a = 1$, $r = -\frac{x-3}{2}$

converges for $|r| = \left|-\frac{x-3}{2}\right| = \left|\frac{x-3}{2}\right| < 1$

$$-1 < \frac{x-3}{2} < 1$$

$$-2 < x-3 < 2$$

$$\{x \mid 1 < x < 5\} = I$$

$$S = \frac{a}{1-r} = \frac{1}{1-\left(-\frac{x-3}{2}\right)} = \frac{1}{1+\frac{x-3}{2}} =$$

$$= \frac{1}{\frac{2+x-3}{2}} = \frac{2}{x-1}$$

what series do you get if you differentiate term-by-term?

$$f(x) = 1 - \tfrac{1}{2}(x-3) + \tfrac{1}{4}(x-3)^2 + \cdots + \left(-\tfrac{1}{2}\right)^n (x-3)^n + \cdots$$

$\Rightarrow$ ANS.

$$\boxed{f'(x) = -\tfrac{1}{2} + \tfrac{1}{2}(x-3) \overbrace{-\tfrac{1}{8}(x-3)^3}{} + \cdots + n\cdot\left(-\tfrac{1}{2}\right)^n (x-3)^{n-1} + \cdots}$$

1 2 3

$$= -\tfrac{1}{2} + \tfrac{1}{2}(x-3) - \tfrac{3}{8}(x-3)^2 + \tfrac{4}{16}(x-3)^3 - \tfrac{5}{32}(x-3)^4 + \cdots$$

$$= \sum_{n=1}^{\infty} \frac{n}{(-2)^n}(x-3)^{n-1} \quad \text{OR} \quad \sum_{1}^{\infty} (-1)^n \frac{n}{2^n}(x-3)^{n-1}$$

$$\text{OR} \sum_{1}^{\infty} n \underbrace{\left(-\tfrac{1}{2}\right)^n}_{} (x-3)^{n-1}$$

$$\left((-1)\left(\tfrac{1}{2}\right)\right)^n = (-1)^n \left(\tfrac{1}{2}\right)^n$$

For what values does this NEW Series converge?

$$f'(x) = -\frac{1}{2} + \frac{1}{2}(x-3) + \cdots + n\cdot\left(-\frac{1}{2}\right)^n (x-3)^{n-1} + \cdots$$

$I = (1,5)$ for $f(x)$

$I = (1,5)$ for $f'(x)$

What is its sum?

$$f'(x) = \sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^n (n)(x-3)^n = -\frac{2}{(x-3)^2}$$

$f(x) = 2(x-3)^{-1}$

$f'(x) = -2(x-3)^{-2}$

$$\int \frac{2\,dx}{x} = 2\ln|x-3| + C$$

So if we "see" $2\ln|x-3|$ as the antiderivative of $\frac{2}{x-3}$, then term-by-term integration gives the power series for $2\ln|x-3|$

$$f(x) = \sum \left(-\frac{1}{2}\right)^n (x-3)^n = \frac{2}{x-3} \longrightarrow$$

$u = x-3$
$du = dx$

$$\int (x-3)^n dx = \frac{(x-3)^{n+1}}{n+1} + C$$

$$\int f(x)\,dx = 2\ln|x-3| + C$$

$$= \sum_{n=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^n (x-3)^{n+1}}{n+1} + \hat{C}$$

$$= 2\ln|x-3| + C$$

#55 Remark: If $f(x)$ has two different power series representations, then they're identical.

Power Series Representation is unique.