§ 10.6 #58

$\sum_{n=0}^{\infty} (-1)^n \frac{1}{n!}$ want error < 5×10^{-6}

$|S - S_n| = \text{Error} = \left| \sum_{k=n+1}^{\infty} (-1)^k \frac{1}{k!} \right| < 5 \times 10^{-6}$

$<$ 1st term we throw away $= |a_{n+1}|$

Want $\frac{1}{n!} < 5 \times 10^{-6}$ $\quad .000005$

$\frac{1}{2}, \frac{1}{3!} = \frac{1}{6}, \frac{1}{4!} = \frac{1}{24}, \frac{1}{5!} = \frac{1}{120}$

$\frac{1}{6!} = \frac{1}{720}, \frac{1}{7!} = \frac{1}{5040}$

$\frac{1}{8!} \approx 2.4 \times 10^{-5}$

$\frac{1}{9!} \approx 2.8 \times 10^{-6} < 5 \times 10^{-6}$ DONE

Theorem 15 says Error $< |(n+1)^{th}$ term$|$

$n+1 = 9$ will work.

So, $\sum_{k=0}^{\infty} (-1)^k \frac{1}{k!} = \sum_{k=0}^{8} (-1)^k \frac{1}{k!} \boxed{+ \sum_{k=9}^{\infty} (-1)^k \frac{1}{k!}}$

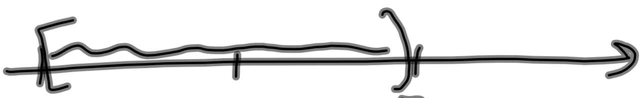
Run n to 8

Can throw it away

Back to § 10.7

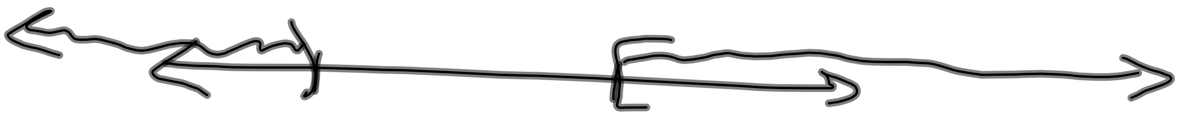
Recall T18 says if $\sum a_n x^n$ converges for $x = 7$, then it converges $\forall\ x \in (-7, 7)$ or $x = -7$

OR



Divergence version

Diverges at $x = 7$, then diverges $\forall\ x\ (-\infty, -7) \cup (7, \infty)$

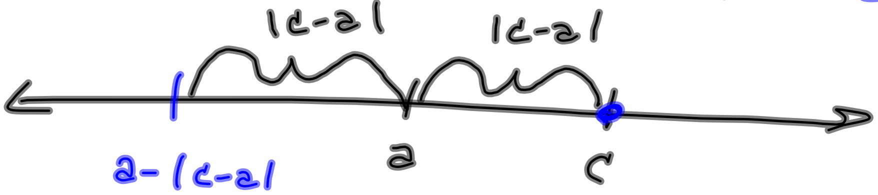


My Corollary to T 13:
Same statement for

$\sum c_n (x-a)^n$ = Power series

centered at $x=a$. If it converges at $x=c$, then it converges $\forall x$ such that $|x-a| < |c-a|$

Assume $c > a$



$= a-(c-a)$
$= 2a-c$

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Example $\boxed{\sum_{n=0}^{\infty} (x-3)^n}$ (geometric) centered

at $x=3$ converges for every $x \ni \boxed{|x-3|<1}$

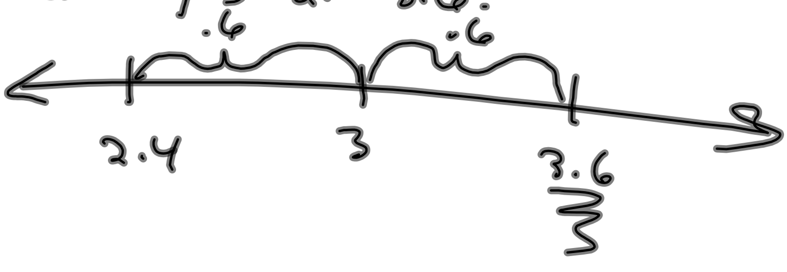
$$\sum_{n=1}^{\infty} a x^{n-1} = \sum_{n=0}^{\infty} a x^n$$

$\to a=1, \; r=x-3$

It converges at $x=2.5$

T18's corollary (My version) says it converges $\forall x \in (2.5, 3.5)$ because it converges at 2.5.

It converges $\forall x \in (2.4, 3.6)$ because it converges at 3.6.



Book's Corollary:

3 possibilities for $\sum c_n (x-a)^n$

1. It has a finite R = radius of convergence & you need to check endpoints.
2. It has $R = \infty$.
3. $R = 0$ (Converges only at $x = a$:

$$\sum c_n (a-a)^n = \sum c_n \cdot 0 = 0$$

Example of 3 $\sum n!\,(x-a)^n$

$$\left| \frac{(n+1)!\,(x-a)^{n+1}}{n!\,(x-a)^n} \right| = (n+1)|x| \xrightarrow{n\to\infty} \infty \text{ for } \underline{\text{any}}\ \underbrace{x \neq 0}_{\text{Fixed}}.$$

(Want $(n+1)|x| > M$

$$n+1 > \frac{M}{|x|}$$

$$n > \frac{M}{|x|} - 1$$

Derek says $\boxed{R = \frac{1}{3}}$

Abs.: $I = (\frac{1}{3}, 1)$ once you get this you're done, because it's not alternating.

Cond.: $I = (\frac{1}{3}, 1)$

Find R. Check endpoints.

Remember, Absolute Convergence means $\sum \left| c_n (x-a)^n \right|$ converges.

Get $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$ & it gives you AN interval. From this, you obtain R. But you have to check endpoints to get THE interval I.

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The tipping point between absolute and conditional convergence is razor thin.

$\sum$ 10.7 I #s 4, 15, 22, 29*, 30*, 35, 36
(Maybe #s 37-40 in class? Please look at them.)

* See 10.3 #s 54, 55
#30 29

#53 Cauchy Condensation

$\sum a_n$ converges iff $\sum 2^n a_{2^n}$ converges.

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges

so does $\sum_{n=1}^{\infty} 2^n \cdot \frac{1}{(2^n)^2}$

$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$

$2 \cdot \frac{1}{2} + 2^2 \cdot \frac{1}{(2^2)^2} + 2^3 \cdot \frac{1}{(2^3)^2} + \dots$

You may use
#54a, 54b, 55a RESULTS

54a $\sum \frac{1}{n \ln(n)}$ diverges

54b $\sum \frac{1}{n^p}$ needs $p > 1$

55a $\int_1^{\infty} \frac{dx}{x (\ln x)^p}$ needs $p > 1$

55b $\sum \frac{1}{n (\ln(n))^p}$ needs $p > 1$