Questions?

§10.6 #57,

§10.4 #29, 7*

Direct Comparison is <u>hard</u>. I'd only use it if the comparison fell in my lap.

$$\sqrt{\frac{n+4}{n^4+4}}$$ Thought Process: $\sqrt{\frac{n}{n^4}} = \sqrt{\frac{1}{n^3}}$

$= \frac{1}{n^{3/2}}$ which converges. So try to find something <u>bigger</u> than $\sqrt{\frac{n+4}{n^4+4}}$ that converges.

(otherwise, look for something <u>smaller</u> that DIVERGES.)

Making a fraction Bigger: $\frac{\text{Bigger}}{\text{smaller}}$

(want bigger) $n+4 < n+n = 2n$ for $n>4$

(want smaller) $n^4+4 > n^4 \quad \forall n$

So $\frac{n+4}{n^4+4} < \frac{2n}{n^4} = \frac{2}{n^3}$ so that

$\sqrt{\frac{n+4}{n^4+4}} < \sqrt{\frac{2}{n^3}} = \frac{\sqrt{2}}{n^{3/2}}$ and we <u>know</u>

that $\sum \frac{\sqrt{2}}{n^{3/2}} = \sqrt{2} \sum \frac{1}{n^{3/2}}$ converges, so

$\sum \sqrt{\frac{n+4}{n^4+4}}$ converges.

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$$\sum \sqrt{\frac{n+4}{n^2+4}} \qquad \sqrt{\frac{n}{n^2}} = \sqrt{\frac{1}{n}} = \frac{1}{n^{1/2}}, \text{ so}$$

we're thinking it'll diverge.

So find something SMALLER than $\sqrt{\frac{n+4}{n^2+4}}$ $\ni$

$\sum$(smaller) diverges

Make $\frac{n+4}{n^2+4}$ smaller $\frac{\text{smaller}}{\text{Bigger}}$

want $n+4$ < something

n^2+4 > . .

$n+4$ is bigger than what?

$n+4 > n$

n^2+4 is smaller than what

$n^2+4 < n^2+n^2 = 2n^2 \ \forall \ n > 2$

So, $\frac{n+4}{n^2+4} > \frac{n}{2n^2} = \frac{1}{2n}$ $\cancel{\ \&\ }$

So, $\sqrt{\frac{n+4}{n^2+4}} > \frac{1}{\sqrt{2}\, n^{1/2}}$

We know $\frac{1}{\sqrt{2}} \sum \frac{1}{n^{1/2}}$ diverges, so

$\sum \sqrt{\frac{n+4}{n^2+4}}$ diverges.

$$\sqrt{\frac{n-4}{n^4+4}}$$ converges

compare it to something bigger

$\frac{n-4}{n^4+4} < \frac{n}{n^4} = \frac{1}{n^3}$ is easy.

$$\frac{A}{B} < \frac{\text{Bigger}}{\text{smaller}}$$

$$\sqrt{\frac{n-4}{n^4-4}} < \sqrt{\frac{n}{n^4-4}} < \sqrt{\frac{n}{n^4-\frac{n^4}{2}}} = \sqrt{\frac{n}{\frac{n^4}{2}}}$$

$= \sqrt{\frac{2}{n^3}}$ etc.

$$\sqrt{\frac{n-4}{n^2+4}} \quad \sqrt{\frac{n-4}{n^2-4}}$$

make $\frac{n-4}{n^2+4} > \frac{n-\frac{1}{2}n}{n^2+n^2} = \frac{\frac{1}{2}n}{2n^2}$

Make $\frac{n-4}{n^2-4} > \frac{\frac{1}{2}n}{n^2} = \frac{1}{2n}$

$$\frac{n+4}{n^2+4} > \frac{n}{n^2+n^2} = \frac{n}{2n^2} = \frac{1}{2n}$$

~~Ex~~ 10.4 #29

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\ln n}$$

$$\lim_{n\to\infty} \frac{\ln n}{\sqrt{n}} = \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{2\sqrt{n}}} = \lim_{n\to\infty} \frac{2\sqrt{n}}{n} = \lim_{n\to\infty} \frac{2}{\sqrt{n}} = 0$$

So $\sqrt{n}$ is eventually bigger than $\ln n$.

If $\sum \frac{1}{\sqrt{n}\sqrt{n}}$ diverges, then so does $\sum \frac{1}{\sqrt{n}\ln n}$

Recall

$$\sum_{n=1}^{\infty} x^{n-1} = \frac{1}{1-x}$$

$1 + x + x^2 + x^3 + \cdots$

$a = 1,\ r = x$

converges $\forall$ $|x| < 1$

E3 $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$ Analyze its convergence

$$\left| \frac{x^{n+1}}{n+1} \cdot \frac{n}{x^n} \right| = \frac{n}{n+1} |x| \xrightarrow{n \to \infty} |x|$$

For convergence, we need $\left| \frac{a_{n+1}}{a_n} \right| < 1$

" " $|x| < \boxed{1 = R}$

R = radius of convergence. $x \in (-1, 1)$

Check endpoints:

$x = 1$: $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{1}{n}$ converges conditionally.

$x = -1$: $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(-1)^1}{n} = \sum_{n=1}^{\infty} \frac{(-1)^{n+2}}{n}$ converges conditionally.

(a) $R = 1$

(b) Absolute convergence on $(-1, 1)$

(c) Conditional convergence on $[-1, 1]$

$\boxed{T18}$ Convergence Thm for Power Series.

If $\sum a_n x^n$ converges at $x = c \neq 0$, then it converges absolutely $\forall\ |x| < |c|$

If it diverges @ $x = d$, then it diverges $\forall\ |x| > |d|$

Ratio Test gives us absolute convergence.

Read Corollary to T18.

R = Radius of Convergence

I = Interval of Convergence.

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#3 $\sum_{n=0}^{\infty} (-1)^n (4x+1)^n$

$\left|\frac{(4x+1)^{n+1}}{(4x+1)^n}\right| = |4x+1| \overset{\text{want}}{<} 1$

$-1 < 4x+1 < 1$

$-2 < 4x < 0$

$-\frac{1}{2} < x < 0$

$\boxed{\begin{array}{l} |4x+1| > 1 \text{ means} \\ 4x+1 > 1 \text{ OR } 4x+1 < -1 \end{array}}$

$|4x+1| < 1$ means

$\underbrace{4x+1 < 1 \text{ AND } 4x+1 > -1}$

$-1 < 4x+1 < 1$

Don't use the shorthand

$-1 > 4x+1 > 1$

(a) $R = \frac{1}{4}$

(b) Converges absolutely for $x \in (-\frac{1}{2}, 0)$

(c) " conditionally for $x \in (-\frac{1}{2}, 0)$ $\therefore$ ___

$x = -\frac{1}{2}$ $\sum (-1)^n (4x+1)^n = \sum (-1)^n (-2+1)$

$= \sum (-1)^n (-1) = \sum (-1)^{n+1}$ Diverges.

$x = 0$ ... $\sum (-1)^n (1)$ Diverges

10.7 I #s 4, 15, 22, 29, 30, 35, 36