$f(x) = x^2 + 1$

Simpson: $\frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$

$\frac{x^3}{3}$

$$\boxed{|E_S| \le \frac{M(b-a)^5}{180n^4} = 0}$$

$f(x) = x^2 + 1$

$f'(x) = 2x$

$f''(x) = 2$

$f'''(x) = f^{(4)}(x) = 0 \equiv M$

Simpson's uses a quadratic approximation

For a quadratic $f(x)$, Simpson's is EXACT.

$g(x) = 2x^3 + \cdots$

$g'(x) = 6x^2 + \cdots$

$g''(x) = 12x$

$g'''(x) = 12$

$g^{(4)}(x) = 0 \equiv M$ for Simpson's !?

$f(x) = x^3$ on $[0, 1]$ $n = 4$

$\left(\frac{1}{4}\right)^3 = \frac{1}{64} = y_1$ $\quad \frac{\frac{1}{4}}{3}\left[0 + 4\left(\frac{1}{64}\right) + 2\left(\frac{1}{8}\right) + 4\left(\frac{27}{64}\right) + 1\right]$

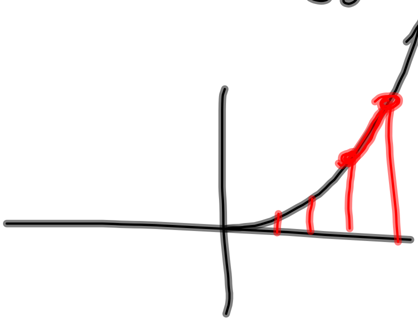
$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ $\quad = \frac{1}{12}\left[\frac{1}{16} + \frac{4}{16} + \frac{27}{16} + \frac{16}{16}\right]$

$\left(\frac{3}{4}\right)^3 = \frac{27}{64}$ $\quad = \frac{1}{12}\left[\frac{48}{16}\right] = \frac{3}{12} = \frac{1}{4}$

$1^3 = 1 = y_4$

$\int_0^1 x^3 dx = \frac{x^4}{4}\Big]_0^1 = \frac{1}{4}$

It nails the cubic!



10.6 Finish   Assume a_n's ≥ 0 $\forall n$

$\boxed{T17}$ Rearranging $\sum a_n$ changes nothing
$\sum a_n = \sum |a_n|$

$\boxed{E6}$ Rearranging a CONDITIONALLY convergent series CAN change the sum.

Think $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$

Doesn't converge if you look @ $\sum |a_n|$

All a_n's ≤ 0, same deal

$\sum a_n = -\sum |a_n|$

Pg 573 : Checklist for breaking these down.

1. Test for Divergence
2. $\sum_{n=1}^{\infty} ar^{n-1}$
3. $\sum \frac{1}{n^p}$
4. $a_n \geq 0$: $\int_N^{\infty} f(x)\,dx$ (No good for alternating)
5. All negative terms? Examine $-\sum a_n$ OR $\sum |a_n|$
6. Alternating Series

10.7

Consider $\sum_{n=1}^{\infty} ar^{n-1} = \sum_{n=0}^{\infty} ar^{n}$

When does it converge?

For what values of r, i.e.?

Need $|r| < 1$

What about $r = 1$? Newp

" " $r = -1$? "

Consider the power series

$$\sum_{n=0}^{\infty} 5x^n = 5\sum_{n=0}^{\infty} x^n = 5\left[1 + x + x^2 + \dots\right]$$

converges for $|x| < 1$

Test it:

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{5x^{n+1}}{5x^n}\right| = |x| < 1$$

Ratio Test says: for convergence.

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$$\boxed{\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \cdots}$$

Test for convergence.

$0! = 1$

$$\binom{6}{3} = C(6,3) = {}_6C_3 = \frac{6!}{(6-3)!\,3!} = \frac{6!}{3!\,3!}$$

$$= \frac{6\cdot 5\cdot 4}{3!} = 5\cdot 4 = 20$$

$$\binom{5}{2} = \frac{5!}{3!\,2!} = \frac{5\cdot 4}{2} = 10$$

$\{a,b,c,d,e\}$

$\{a,b\}$, $\{a,c\}$, $\{a,d\}$, $\{a,e\}$

$\{b,c\}$, $\{b,d\}$, $\{b,e\}$

$\{c,d\}$, $\{c,e\}$

$\{d,e\}$

$$\binom{6}{0} = \frac{6!}{6!\,0!} = \frac{1}{1} = 1 = \binom{6}{6}$$

1

1 1

1 2 1

This is 1 3 3 1

why $\binom{n}{k}$ is called
the binomial coefficient!

$$\binom{2}{0} \binom{2}{1} \binom{2}{2}$$

$$\binom{3}{0} \binom{3}{1} \binom{3}{2} \binom{3}{3}$$

$$(x+1)^3 = 1x^3 + 3x^2 + 3x + 1$$

$$= \binom{3}{0}x^3 + \binom{3}{1}x^2 + \binom{3}{2}x + \binom{3}{3}$$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y^1 + \cdots + \binom{n}{k}x^k y^{n-k}$$

$$+ \cdots + \binom{n}{n}x^0 y^n$$

$$= \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x \quad !!!$$

Convergence : $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n}\right|$

$= \left|\frac{x}{n+1}\right| = \frac{|x|}{n+1} \xrightarrow{n \to \infty} 0 \overset{\text{want}}{<} 1$

Regardless of x, this thing goes to zero!

Converges for ANY REAL x!

$$\frac{x^n}{n!} \longrightarrow 0$$

Term-by-term differentiation

$$f(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$$

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$$f'(x) = 0 + 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = f(x)$$

what $f(x)$ has this property?

$$f'(x) = f(x)$$

$$y' = y$$

$$\int \frac{y'}{y} = \int 1 \qquad \text{i.e.} \qquad \int \frac{dy}{y} = \int dx$$

$$e^{\ln y} = e^{x+C}$$

$$y = e^{x+C} = \boxed{ke^x}$$

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$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x^{n+1}}{n+1}\cdot\frac{n}{x^n}\right| = \left|\frac{n}{n+1}\right||x| \xrightarrow{n\to\infty} |x|$$

converges for $|x|<1$

what about $x=1$?

$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$ Yes.

what about $x=-1$?

$-1-\frac{(-1)^2}{2}-\frac{1}{3}-\frac{1}{4}-\frac{1}{5}-\cdots$ No!

Converges on $I=(-1,1]$

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