$$\sum_{1}^{\infty} \sqrt{\frac{n+4}{n^4+4}}$$

$\boxed{\text{Should converge, thinking p-test, } n = \frac{3}{2}}$

$< n \ \forall n \geq 5$

eventually

$$\frac{n+4}{n^4+4} \leq \frac{n+4n}{n^4+4} = \frac{5n}{n^4+4} \leq \frac{5n}{n^4}$$

$$= \frac{5}{n^3}$$ would work

$$\frac{n+4}{n^4+4} \leq \frac{n+n}{n^4+4} \leq \frac{2n}{n^4} = \frac{2}{n^3}$$

$n > 4$

Also works.

Little better

More than we need.

Basically make it bigger.

If that bigger converges, good to go.

$$\sqrt{\frac{n-5}{n^{3.2}+7}} \leq \sqrt{\frac{n}{n^{3.2}}} = \sqrt{\frac{1}{n^{2.2}}}$$

$$\sum_{1}^{\infty} \frac{\sin^2 n}{2^n}$$

$$\frac{\sin^2 n}{2^n} \leq \frac{1}{2^n}$$

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \sum_{n=1}^{\infty} \frac{1}{2} \cdot \frac{1}{2^{n-1}} = \frac{1}{2} \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} \quad a=1 \quad r=\frac{1}{2}$$

$$= \frac{1}{2} \cdot \frac{1}{1-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{\frac{1}{2}} = 1 \qquad \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{n-1} \quad a=\frac{1}{2} \quad r=\frac{1}{2}$$

$$= \frac{\frac{1}{2}}{1-\frac{1}{2}}$$

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$$\sum_{n=3}^{\infty} \frac{1}{\ln(\ln(n))}$$

grows slower than n

grows slower than n (something slower than n)

Compare to $\frac{1}{n}$ in the limit.

$$\lim_{n\to\infty} \boxed{\frac{\frac{1}{n}}{\frac{1}{\ln(\ln(n))}}} = \lim \frac{\ln(\ln(n))}{n} \overset{l'H}{=} \lim \frac{\frac{\frac{1}{n}}{\ln(n)}}{1}$$

$$= \frac{1}{n\ln(n)} \xrightarrow{n\to\infty} 0$$

$\frac{1}{n}$ shrinks faster than $\frac{1}{\ln(\ln(n))}$

$\sum \frac{1}{n}$ diverges $\Rightarrow \sum \frac{1}{\ln(\ln(n))}$ diverges.

$$\frac{d}{dn}\ln(f(n)) = \frac{f'(n)}{f(n)} = \frac{\frac{1}{n}}{\ln(n)}$$

T 15 S 10.6 $n+1$ th term is bigger than the remainder $|S - S_n|$

Furthermore $S - S_n$ has same sign as 1st unused term.

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n} - \sum_{n=1}^{10}(-1)^{n+1}\frac{1}{n}$$

$$= \frac{1}{11} - \frac{1}{12} + \frac{1}{13} - \frac{1}{14} + \cdots > 0$$

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③ $\sum (-1)^{n+1} \frac{1}{n 3^n}$ converges ABSOLUTELY

without the help of $(-1)^{n+1}$

⑥ $\sum (-1)^{n+1} \frac{n^2+5}{n^2+4}$

Diverges by Test for Divergence.

$$\frac{n^2+5}{n^2+4} \xrightarrow{n\to\infty} 1 \neq 0$$

⑧ $\sum_{n=1}^{\infty} \frac{10^n}{(n+1)!} (-1)^{n+1}$

$$\frac{10 \cdot 10 \cdot 10 \cdots\cdots 10}{(n+1)(n)\cdot(n-1)\cdots\cdots(1)}$$

Test for divergence?

$$\frac{10}{103} \; \frac{10}{102} \; \frac{10}{101} \cdot \frac{10}{100} \cdot \cdots \cdot \frac{10 \cdot 10 \cdot 10 \cdots 10}{10 \cdot 9 \cdot 8 \cdots 1}$$

$$\frac{10^n}{(n+1)!}$$ Test for Divergence.

$$\lim_{n\to\infty} \frac{10^n}{(n+1)!} = \frac{\infty}{\infty}$$

$$\lim_{n\to\infty} \frac{\frac{1}{(n+1)!}}{\frac{1}{10^n}} = \frac{0}{0}$$ L'Hopital's?

But $(n+1)!$ kicks my butt when I take the derivative.

$$\frac{d}{dn}\left[n(n-1)(n-2)\cdots(3)(2)(1)\right]$$

$= (n-1)! + n\cdot 1\cdot(n-2)! + n\cdot(n-1)\cdot 1\,(n-3)!$

Dang!

Josh Says Ratio Test. Yes.

$$\frac{a_{n+1}}{a_n} = \frac{10^{n+1}}{(n+2)!}\cdot\frac{(n+1)!}{10^n} = a_{n+1}\cdot\frac{1}{a_n}$$

$$= \frac{10}{n+2} \xrightarrow{n\to\infty} 0<1 \Rightarrow \sum\frac{10^n}{(n+1)!} \text{ converges}$$

ABSOLUTELY

$n!$ grows faster than b^n (b constant)

n^n .. .. .. $n!$

n .. .. .. $\ln(n)$

$$\frac{1}{1-x} = \sum_{n=1}^{\infty} x^{n-1} \quad \text{converges for } |x| < 1$$

$$\sum_{n=1}^{\infty} x^{n-1} \qquad \begin{array}{l} a = 1 \\ r = x \end{array}$$

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=1}^{\infty} (-x)^{n-1}$$

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{n=1}^{\infty} (-x^2)^{n-1}$$